Inventory routing

and on-line inventory routing

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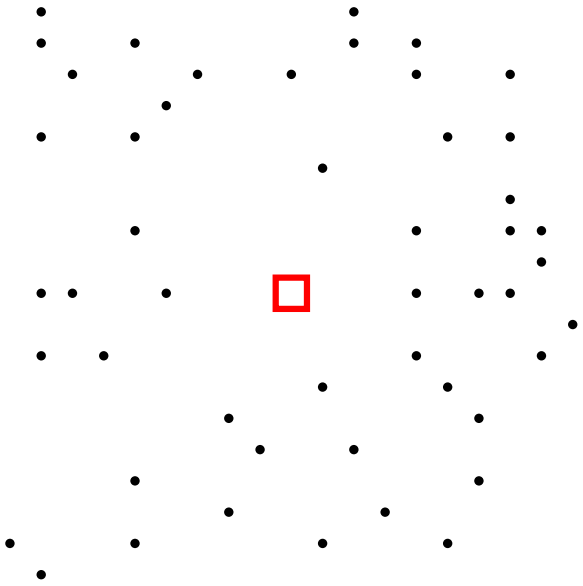
Joint work with M.J. Geiger

The Inventory Routing Problem (IRP)

- One depot
- A set of customers



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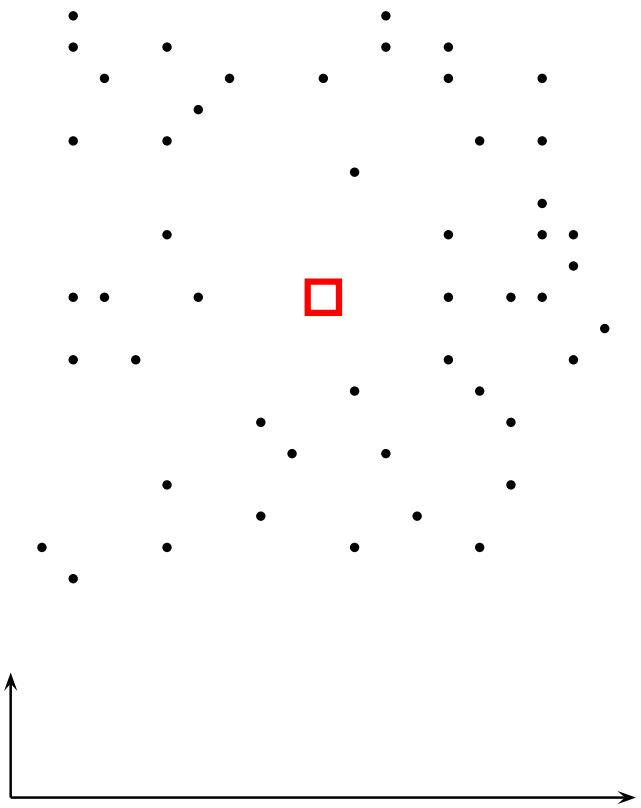

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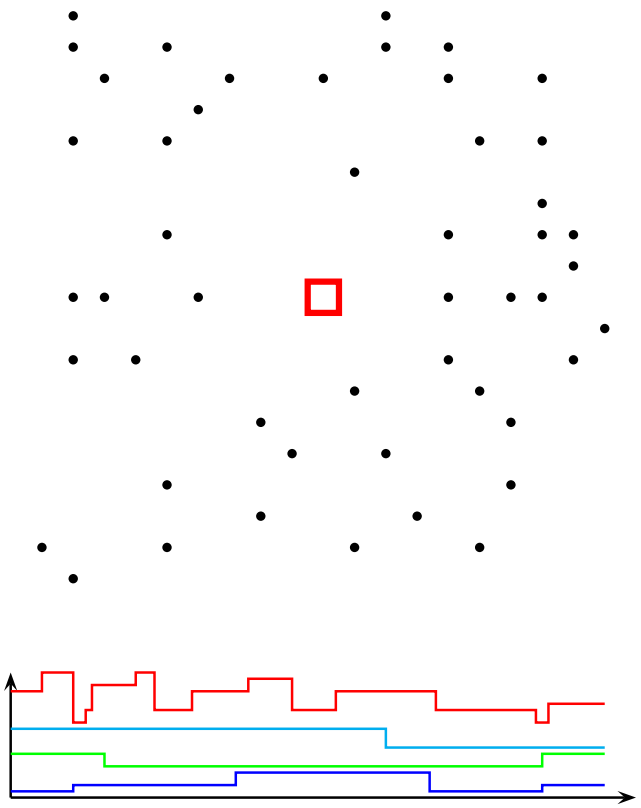

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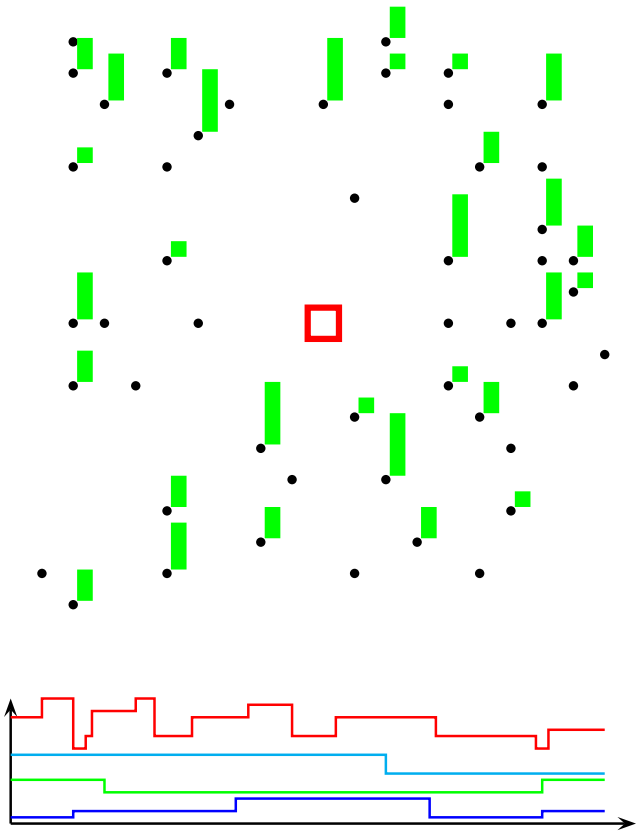

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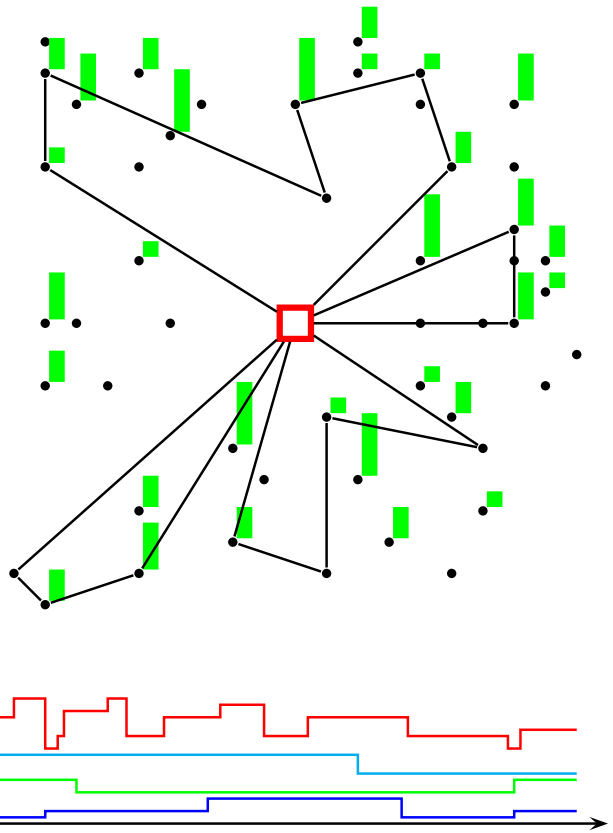

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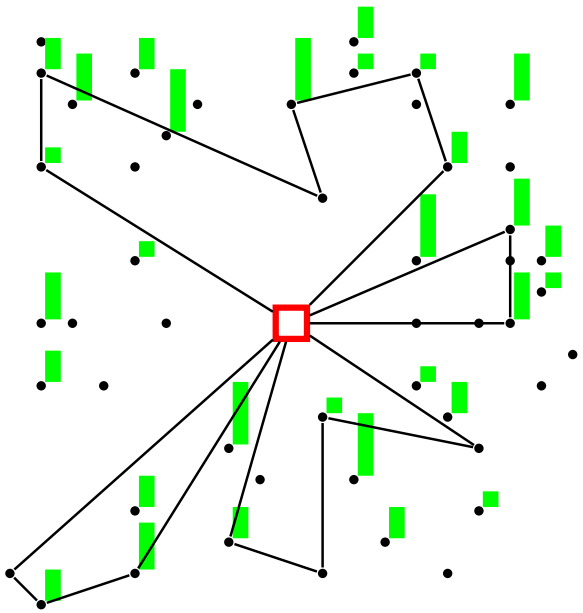

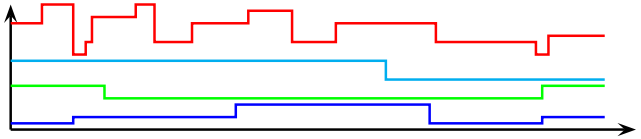

IRP is a real bi-objective optimization problem

Important decisions

For solving the IRP, we must make the following decisions:

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All these decisions are linked together:

- increase delivery quantities $\rightarrow$ change routes or frequency
- change frequency $\rightarrow$ adapt delivery quantities
- ...

Important references

First papers

[Bell *et al.*, 1983]
Improving the distribution of industrial gases with and on-line computerized routing and scheduling optimizer
Interfaces 13(6):4–23, 1983.

[Federgruen and Zipkin, 1984]
A combined vehicle routing and inventory allocation problem
Operations Research 32(5):1019–1037, 1984

Recent surveys

[Bertazzi *et al.*, 2008]
Inventory routing
in *The Vehicle Routing Problem: Latest Advances and New Challenges* 49–72, 2008.

[Cordeau *et al.*, 2011]
Short-haul routing
in *Handbooks in Operations Research and Management Science: Transportation*. To appear.

IRP variants

There exists a large number of variants

- planning horizon (finite/infinite)
- inventory costs and capacities
- production/demand rates (of single/multiple product)
- specific restrictions/regulations
- deterministic/stochastic demand/production
- initial inventory
- fleet (homogeneous/heterogeneous)

And different objectives

- usually a combination of inventory level and routing cost

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What does mean Practical IRP?

- Main goal: helping companies where IRP is important
- Propose a simple output

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- Give different alternatives

A major assumption in our work

To follow a common strategy in companies dealing with IRP
We have separated the decisions

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$$\boxed{\text{Determine quantities for each period}}$$

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Compute best routing for each period

The choice of policies

Standard policies:

DD Day-to-day delivery policy
If not enough in stock, deliver the missing demand

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Any quantity less than the maximum level
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Assumptions

- Serve only if current inventory is not enough
- If served, only full number of period demands

The frequency policy encoding

Solutions are modeled as a frequency f of the deliveries for each customer

$f = 1$	$\rightarrow$	DD policy
$f = 2$	$\rightarrow$	serve for the next two consecutive periods
$\dots$	$\rightarrow$	$\dots$
$f = k$	$\rightarrow$	serve for the next k consecutive periods
$\dots$	$\rightarrow$	$\dots$
$f = +\infty$	$\rightarrow$	OU policy

The policies are simple to understand (even for companies) and easy to encode for us (vector of integer)

Evaluation of solutions

Each solution is measured with the two criteria

Inventory cost

Sum of all inventory levels at customers' at the end of each period. This can be computed in $\mathcal{O}(np)$

Routing cost

Sum of all distances run by the trucks at every period. Solve a VRP for each period. This is a $\mathcal{NP}$-hard problem!!!

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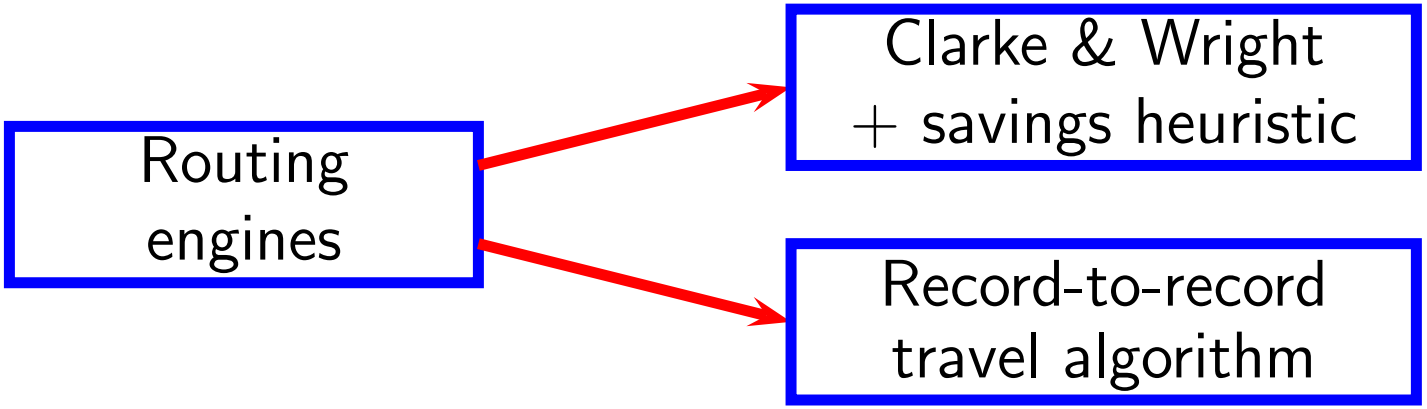

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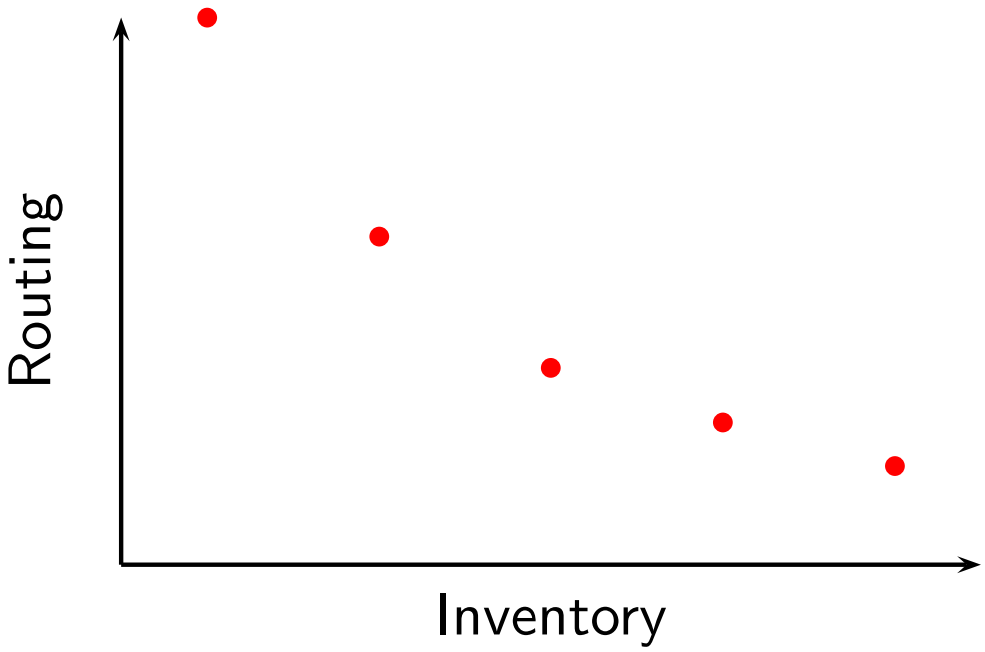
- Identical frequency: f is the same for each customer



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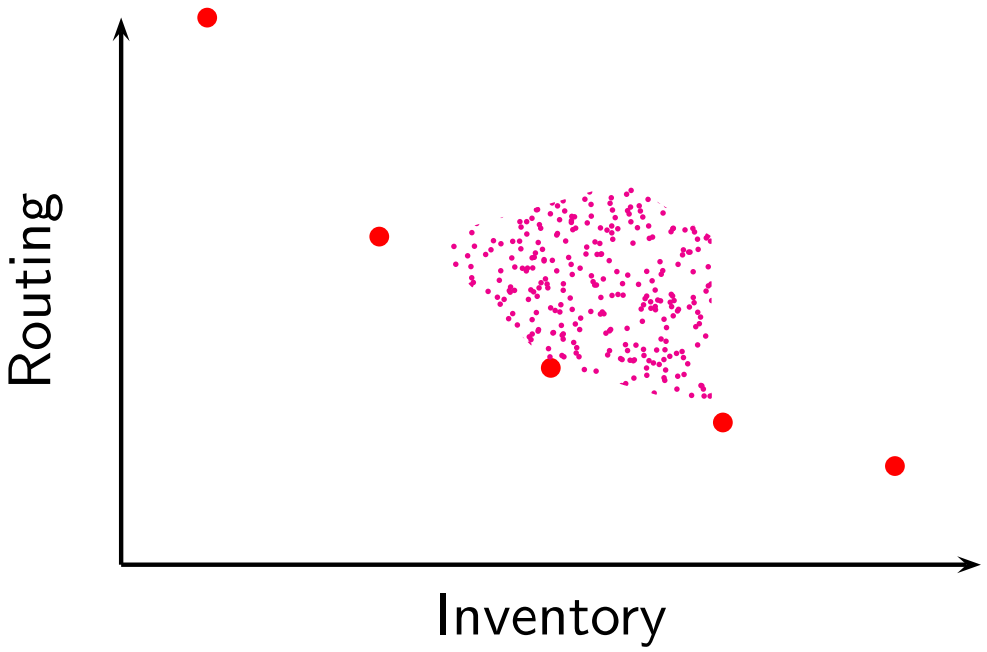

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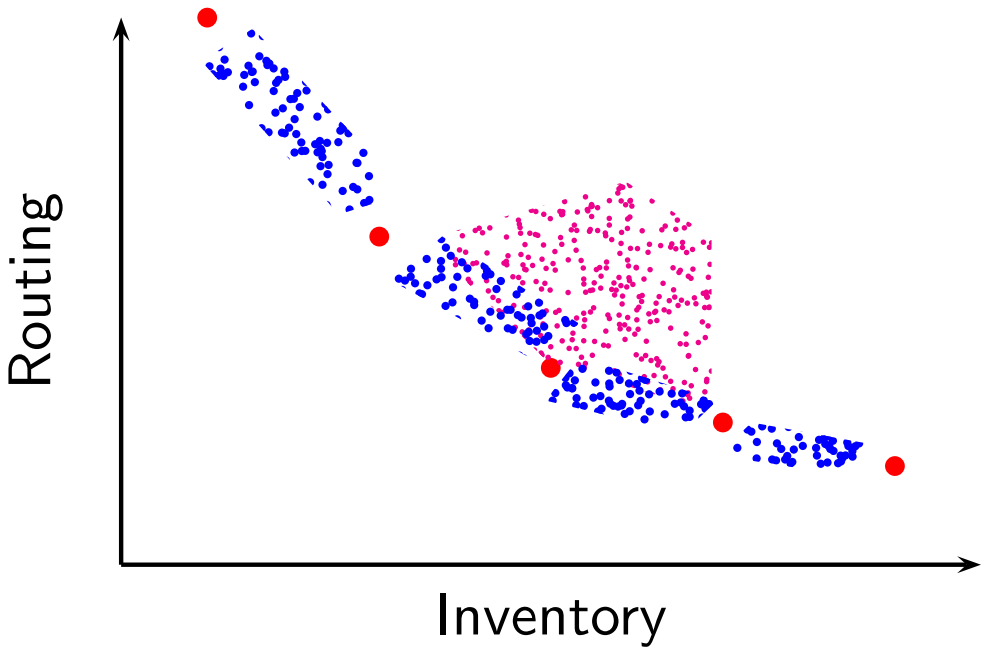
- Totally random: f is chosen at random for each customer
- Controlled random: f is chosen at random between two bounds



Neighborhood

Encoding is a vector of integers
Many moves are possibles but think of the neighborhood size

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s	2	3	1	5	1	$\cdots$	2

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Move 1: Change frequency by ± 1 (here $+1$)

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Move 2: Change frequency by $\pm k$ (here -3)

s_2	2	3	2	2	1	$\cdots$	2

General algorithm

Algorithm 1: MOEA-IRP

Initialization: create an initial population
    → Identical frequency + controlled random
Cleanup: remove dominated solutions
repeat
    Improve all solutions with a local search
        → steepest descent algorithm with Move 1
    Rebuild archive with non-dominated solutions only
until *no more improvements*

Other possible options

- NSGA-II or SPEA
- Path Relinking with elite population

Problem: the *gap* effect

Visualizing the Pareto front, we noticed important gaps

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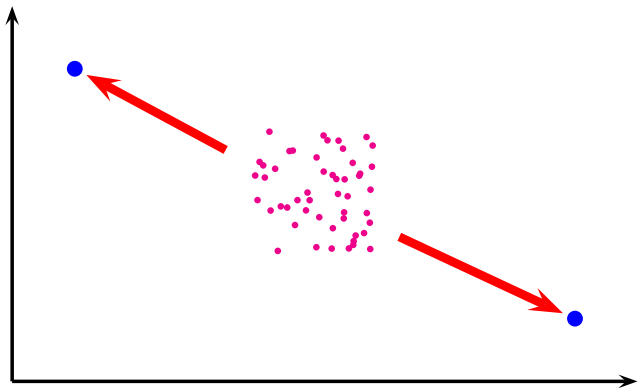


Potential solutions:

- the more controlled random strategy
- the controlled local search strategy

Problem: the *swing* effect

In the IRP-solver we developed, we noticed repeating identical patterns

- Customers are groups together because they run out of goods at the same time
- No cleaver grouping strategy
- Customers may be served together even if they are far away from each others

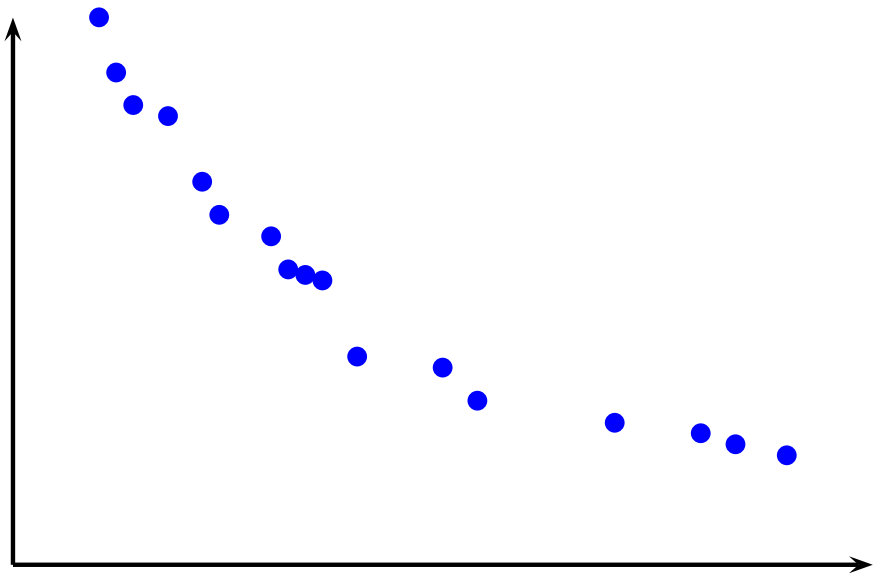
Potential solutions:

- the double frequency encoding
- the binary encoding

Problem: the *number* of solutions

- This approach generates too many solutions
  $\rightarrow$ thousands of solutions, how to handle them?

Implemented solution: the reference set strategy



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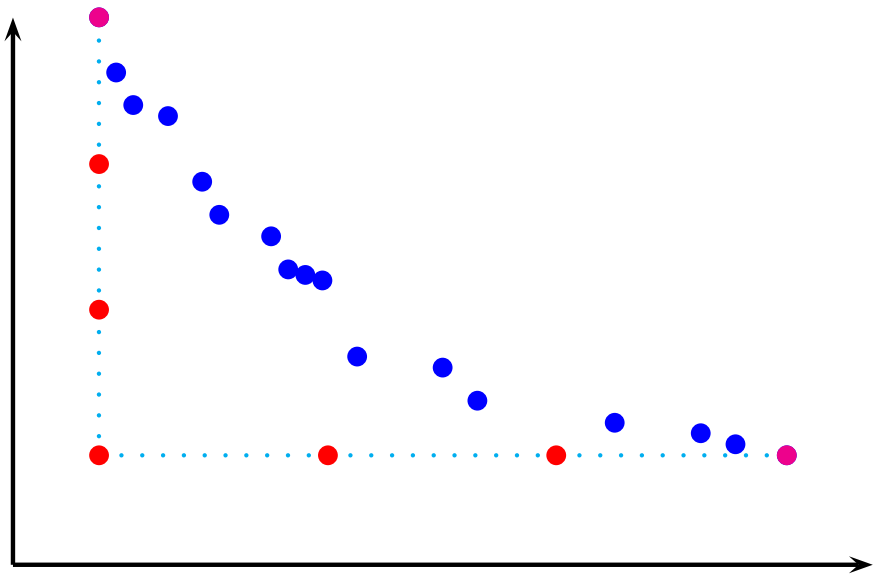

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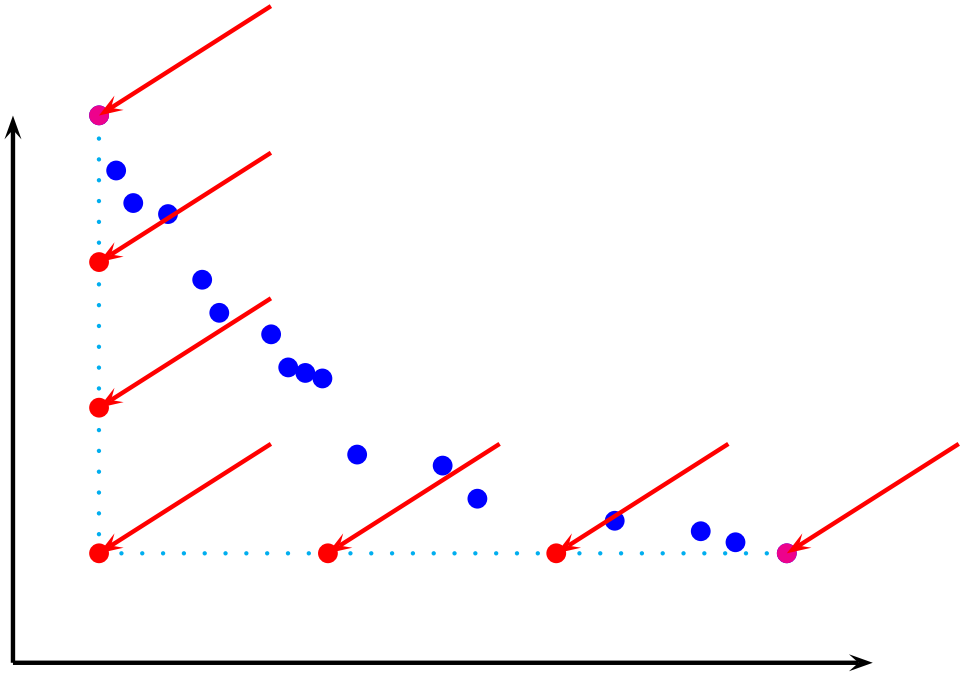


Numerical experiments

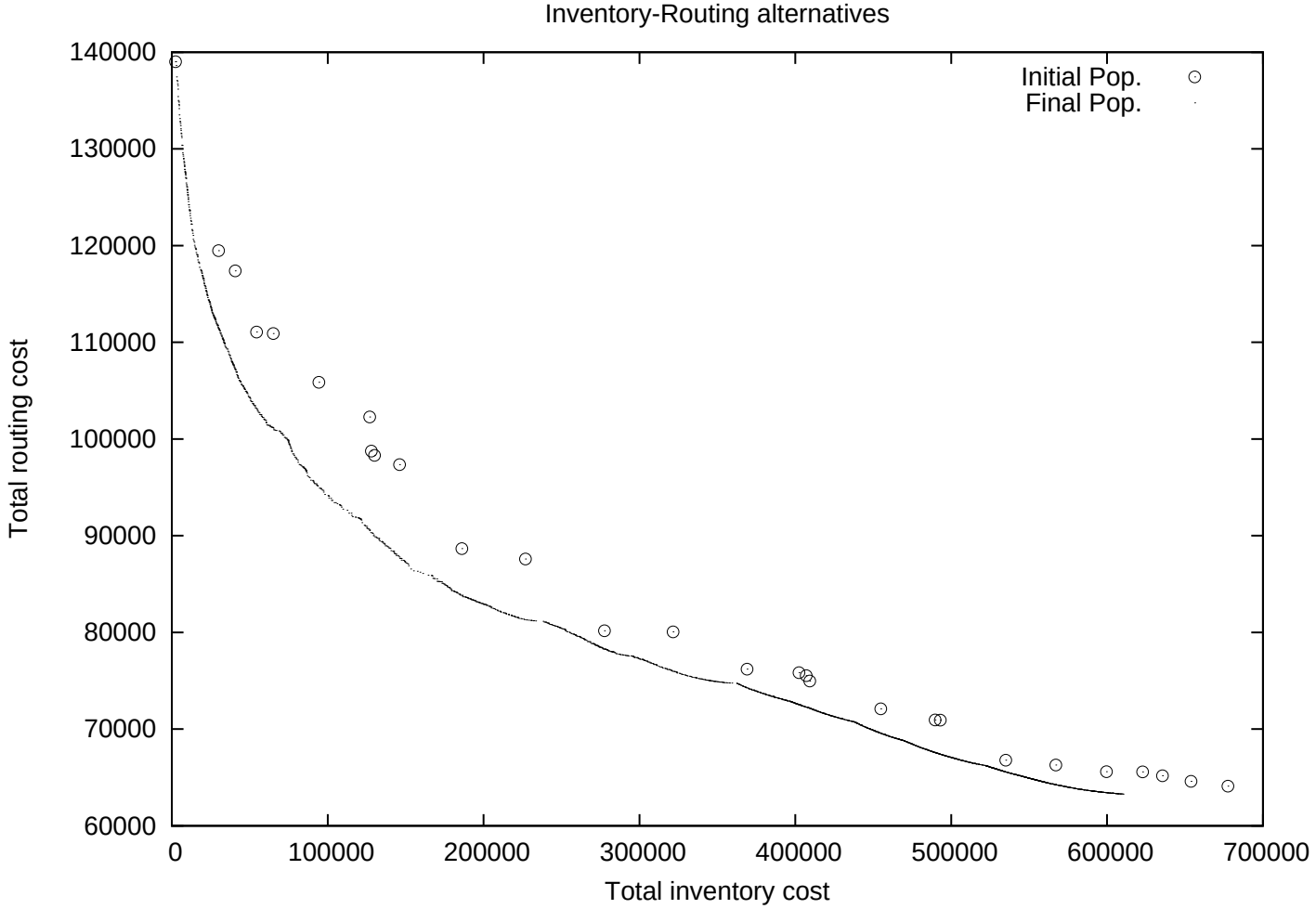
A new set of instances has been generated
inspired from Christofides, Mingozzi and Toth VRP instances

Parameters	**Possible values**
# customers, n	$\{50, 75, 100, 120, 150, 200\}$
Horizon, p	$\{240\}$
Truck capacity, K	$\{140, 160, 200\}$
Demand	{Constant, Increasing, Sinus}

This form a new set of 42 instances
available at http://logistik.hsu-hh.de/IRP

Results (too many solutions)

Output for an instance with 50 customers (2485 alternatives)



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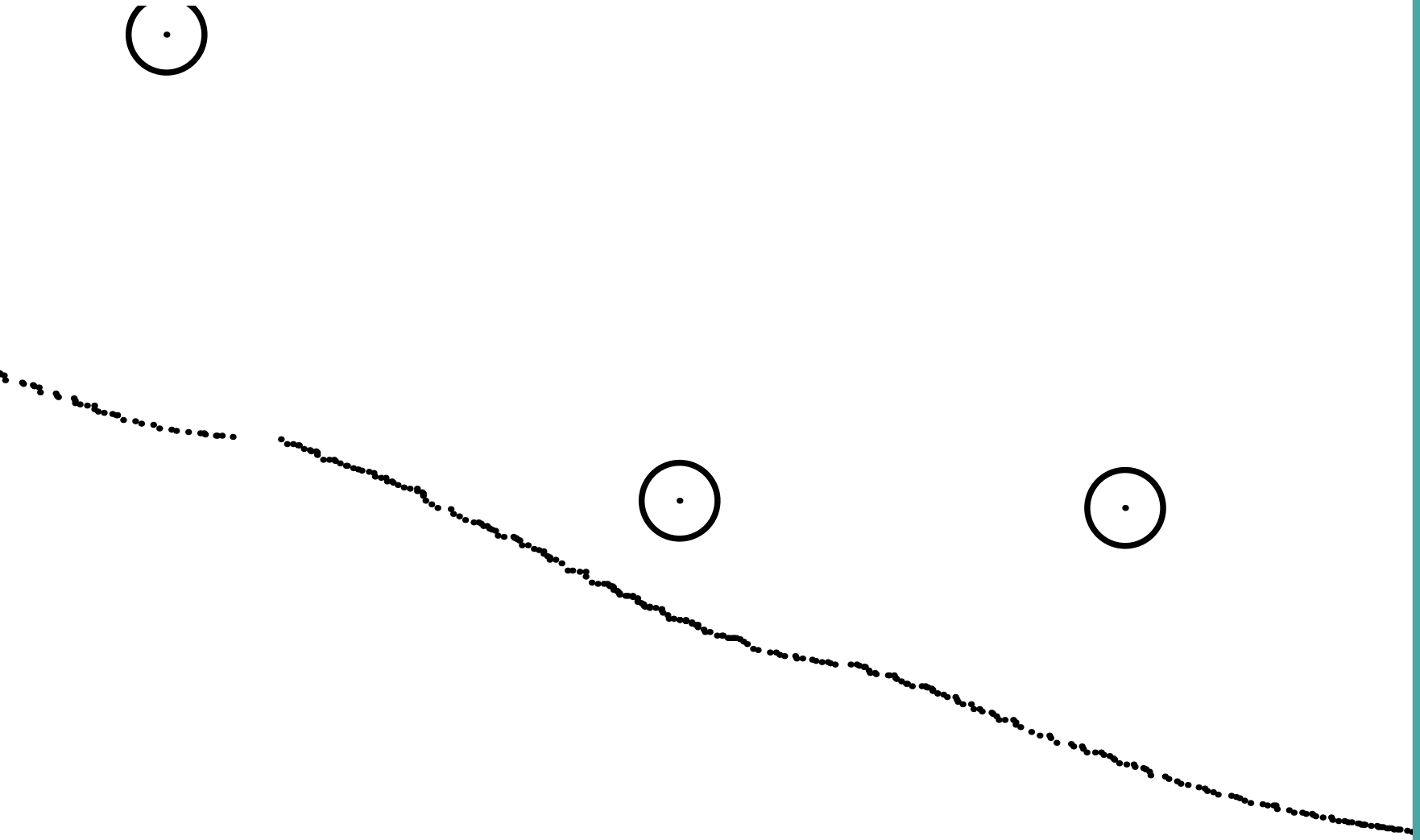

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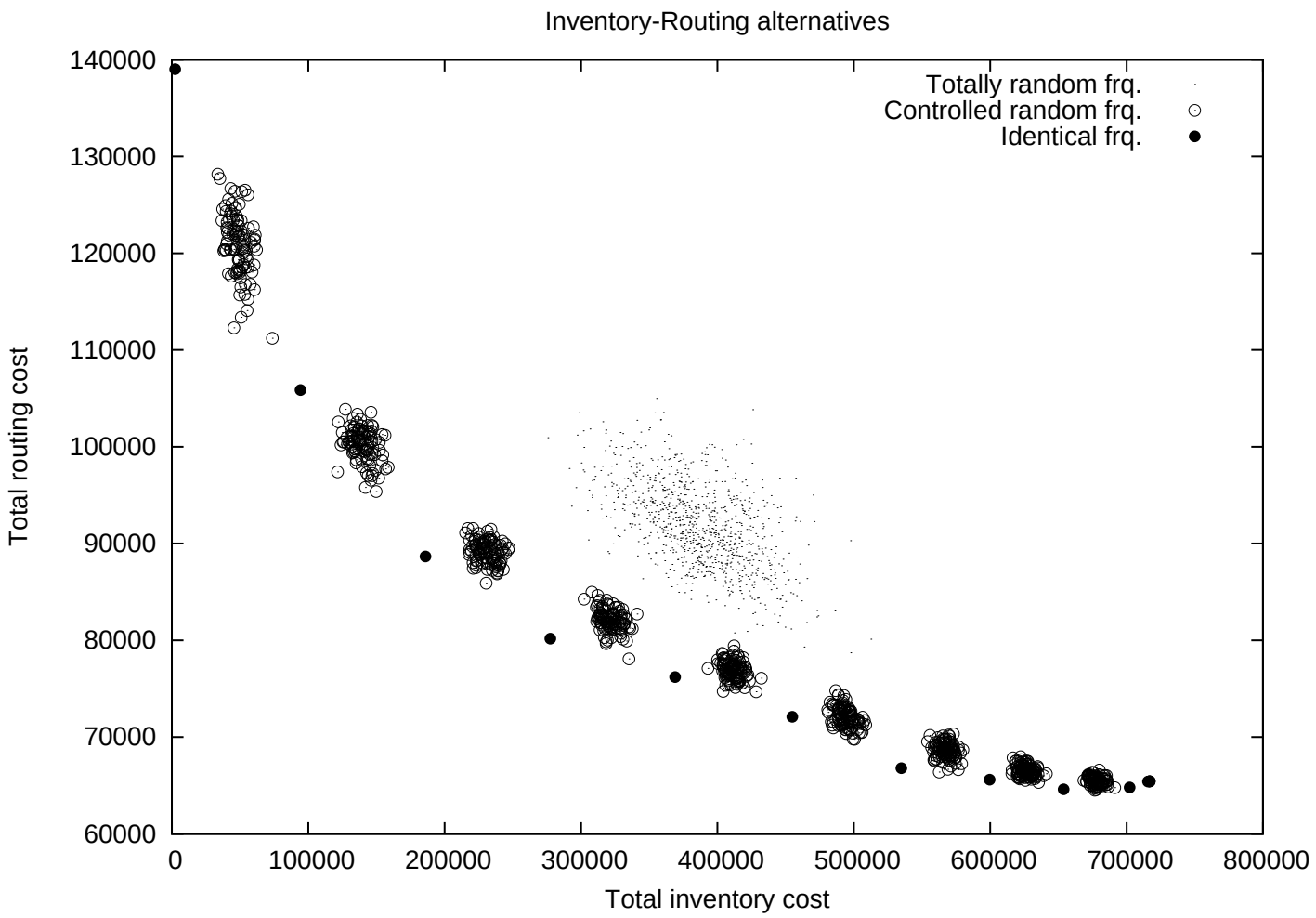

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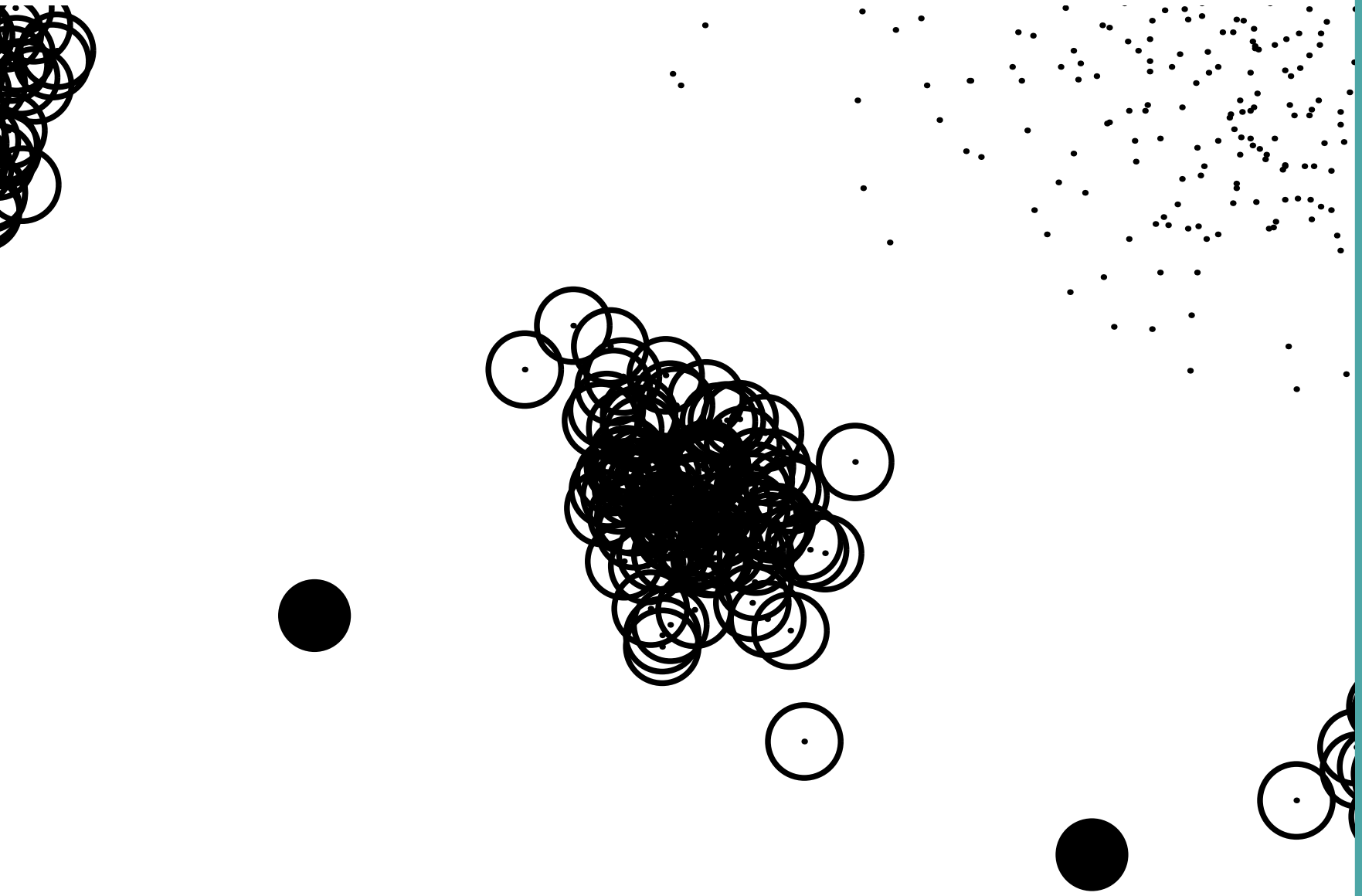

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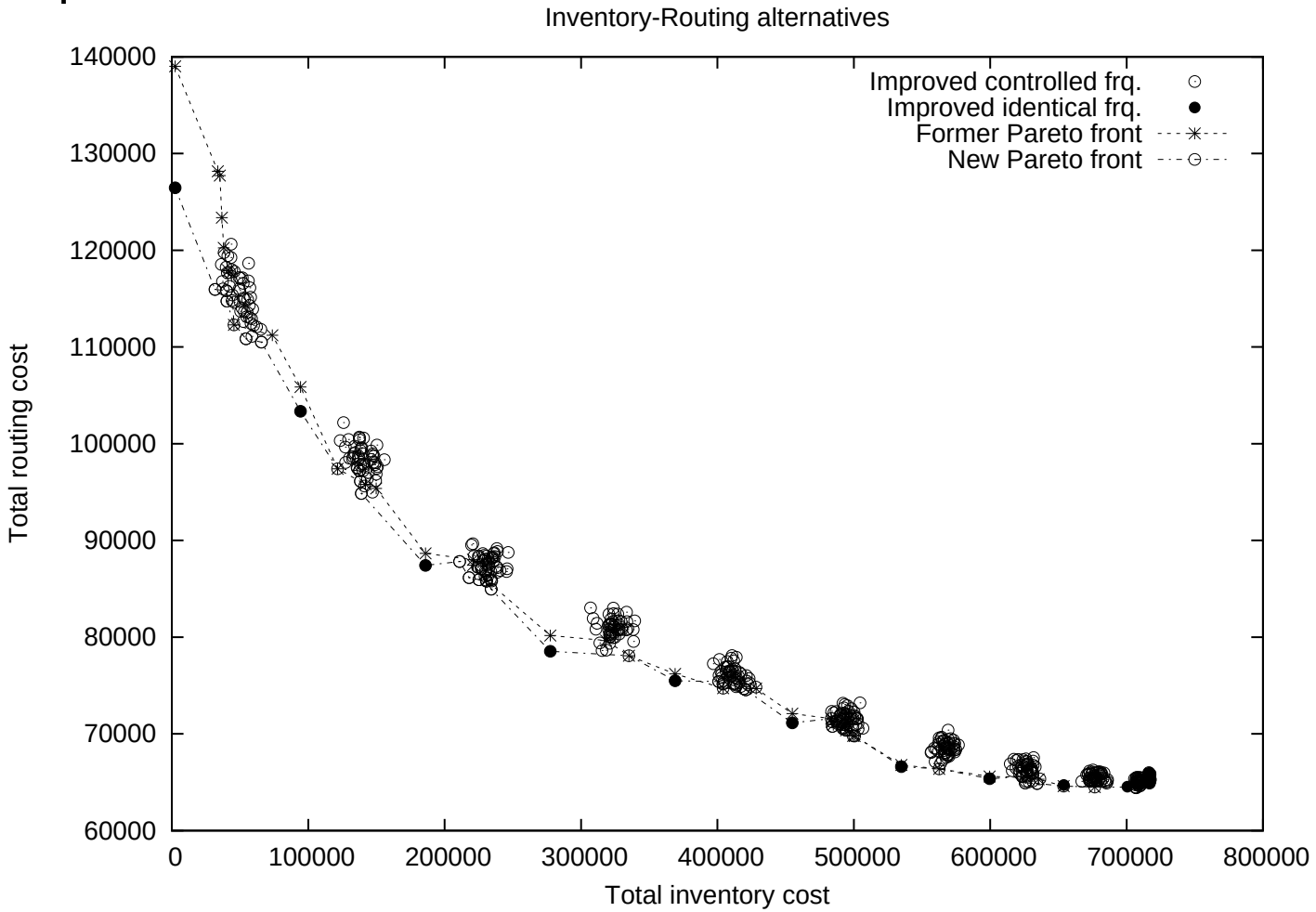

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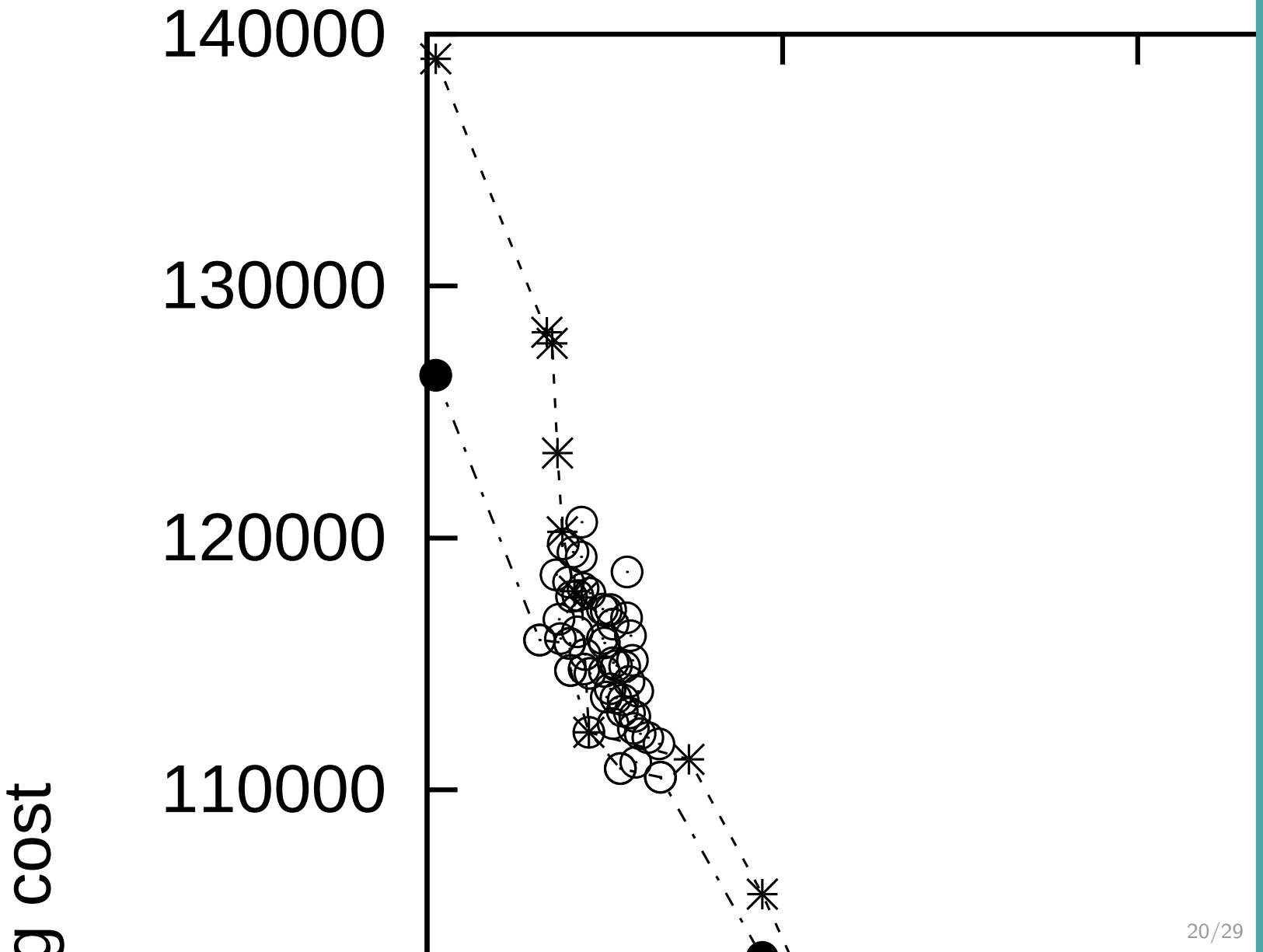

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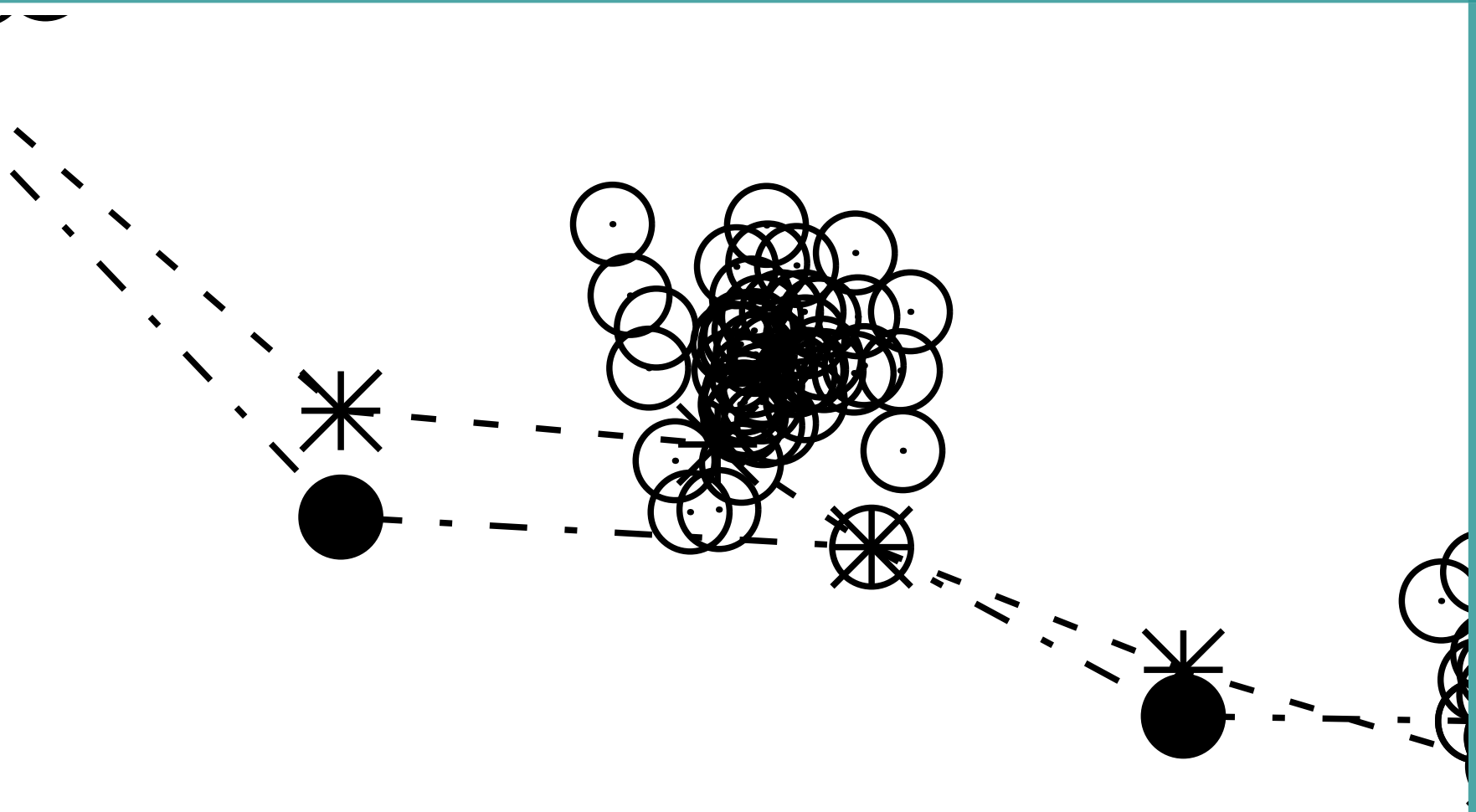
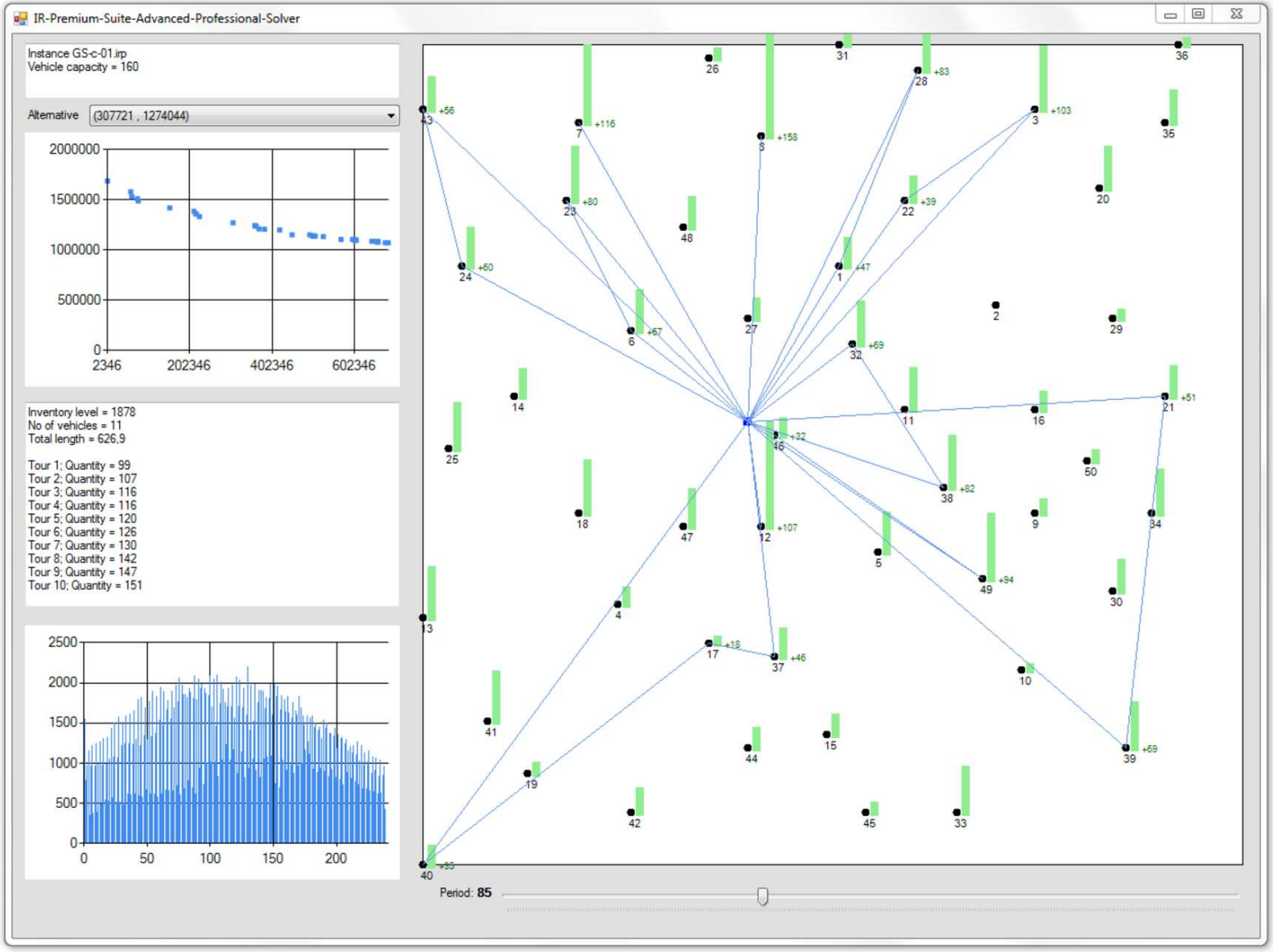
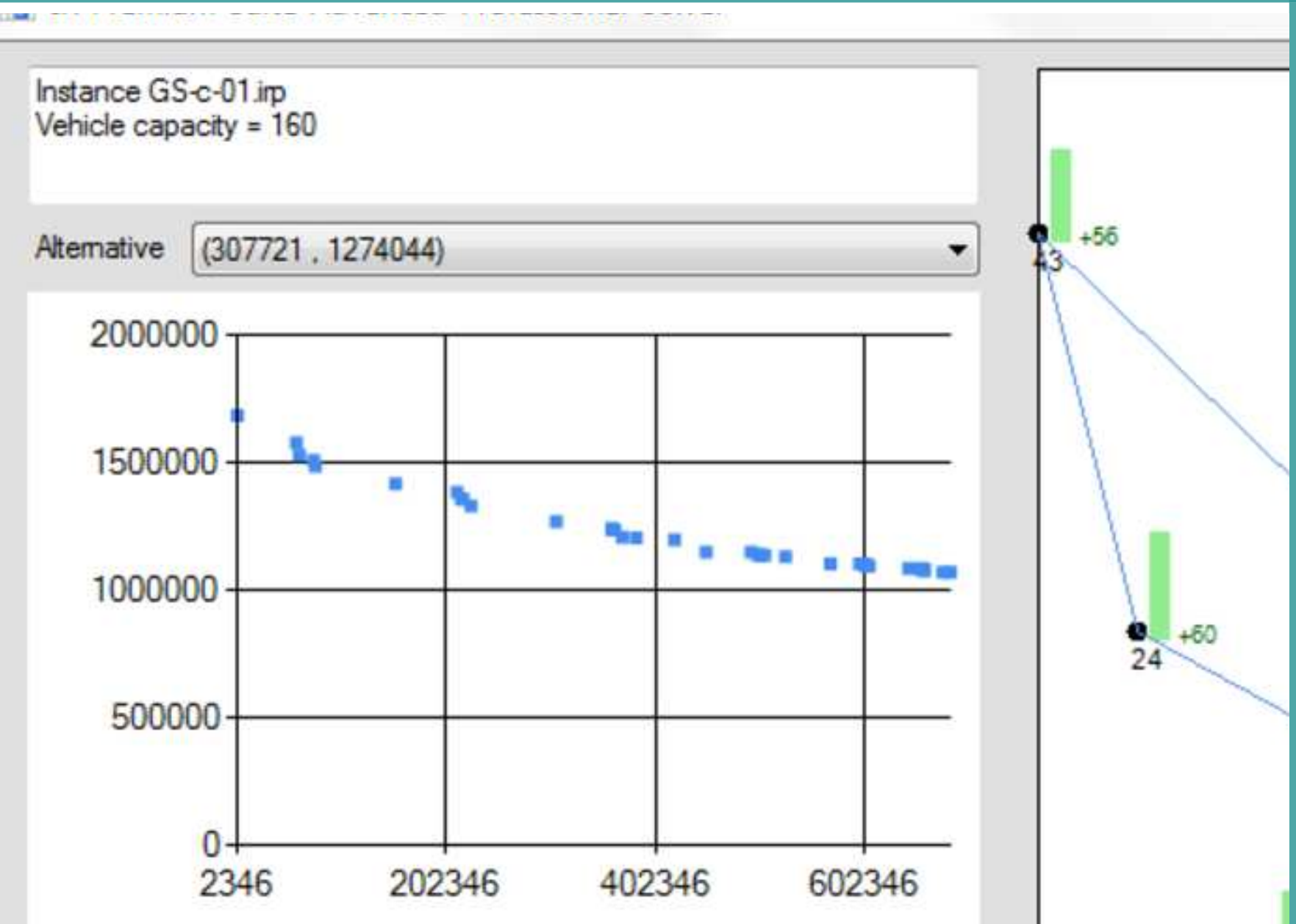


The Inventory Routing Solver



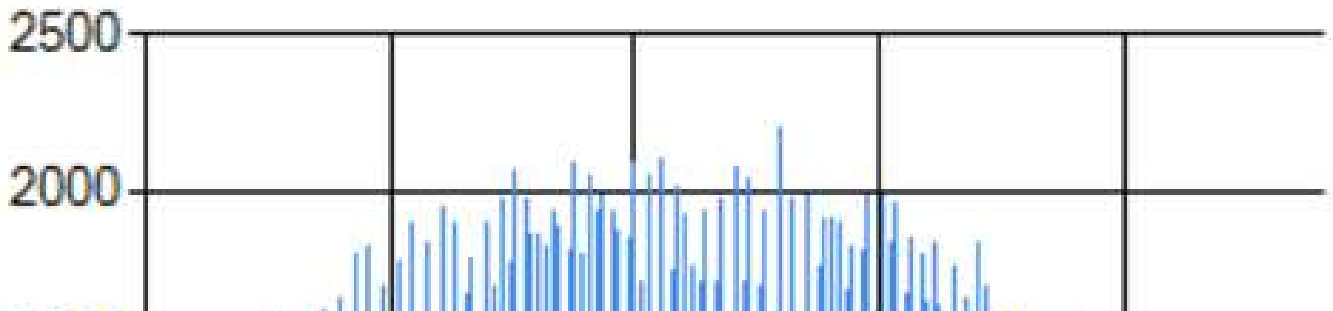
The Inventory Routing Solver (alternatives)

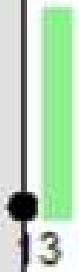


The Inventory Routing Solver (Cost details)

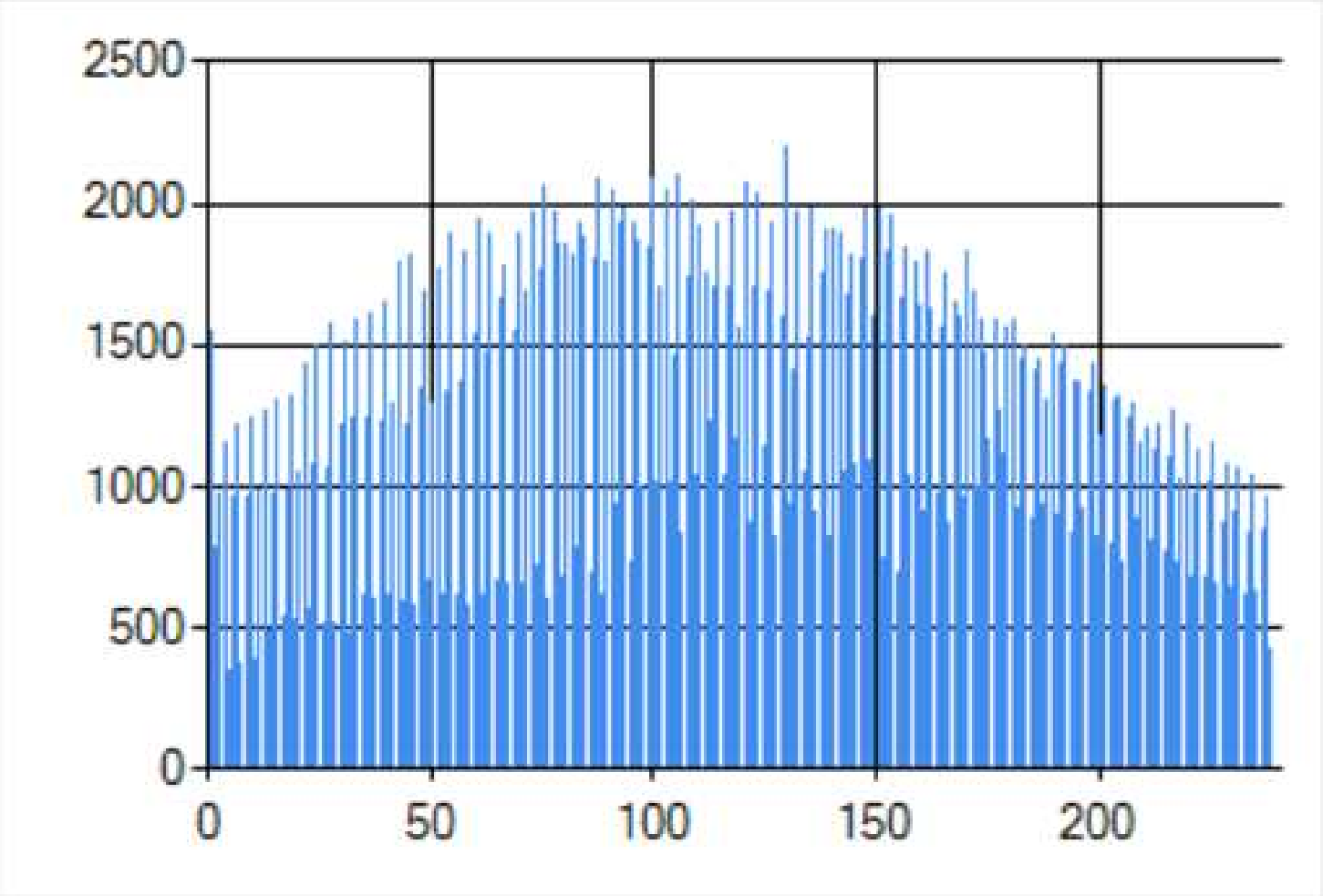
Inventory level = 1878
No of vehicles = 11
Total length = 626,9

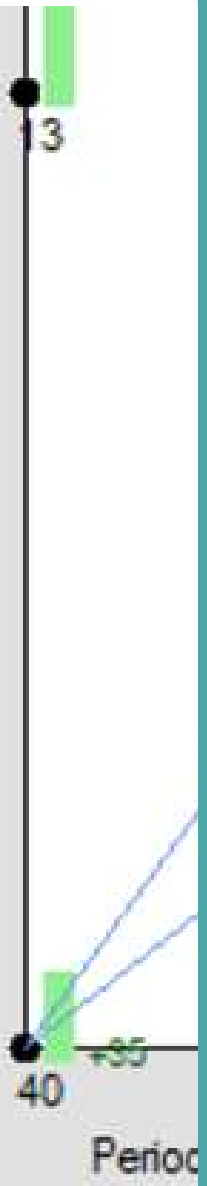
Tour 1; Quantity = 99
Tour 2; Quantity = 107
Tour 3; Quantity = 116
Tour 4; Quantity = 116
Tour 5; Quantity = 120
Tour 6; Quantity = 126
Tour 7; Quantity = 130
Tour 8; Quantity = 142
Tour 9; Quantity = 147
Tour 10; Quantity = 151



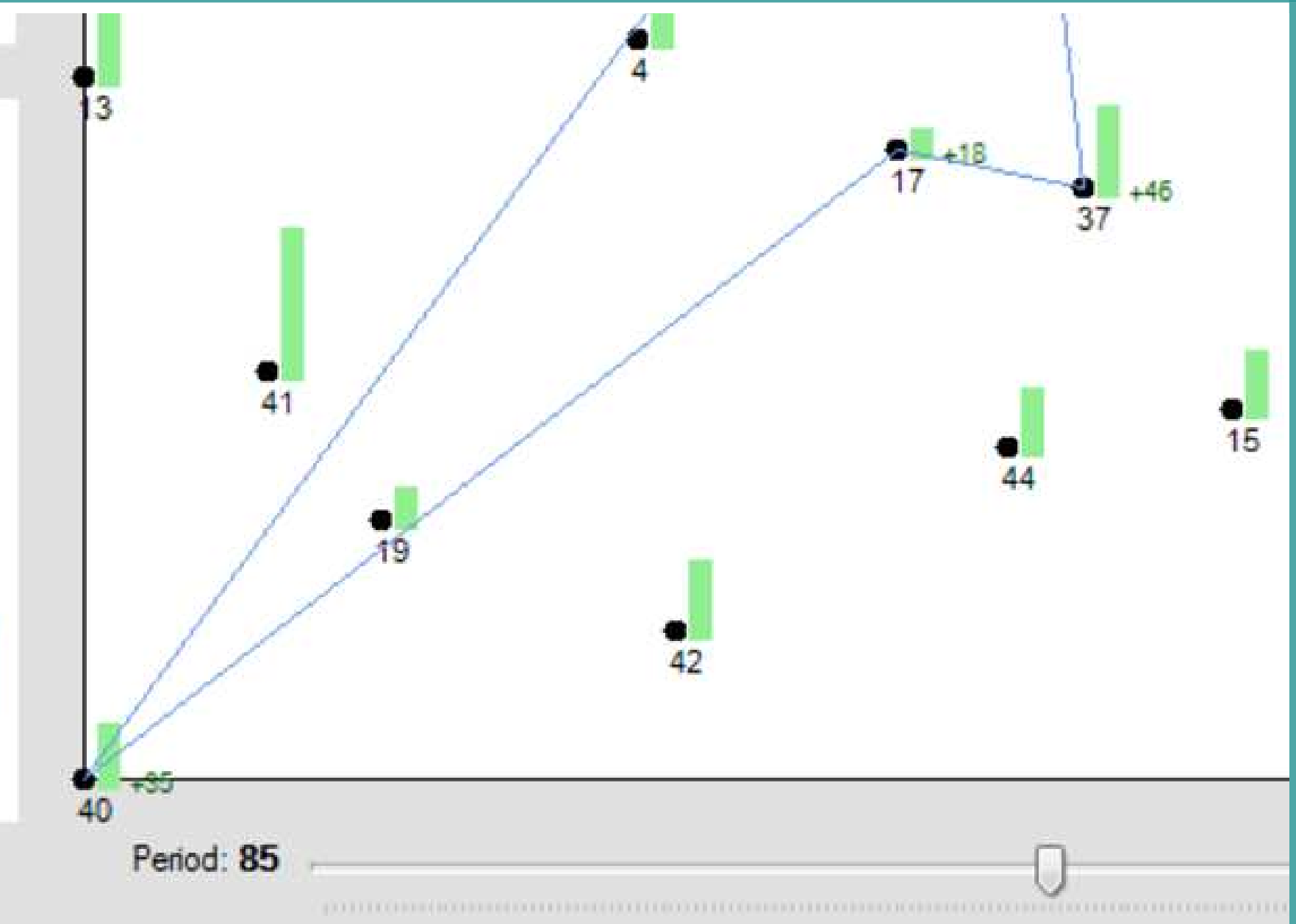


The Inventory Routing Solver (Inventory)





The Inventory Routing Solver (Periods)



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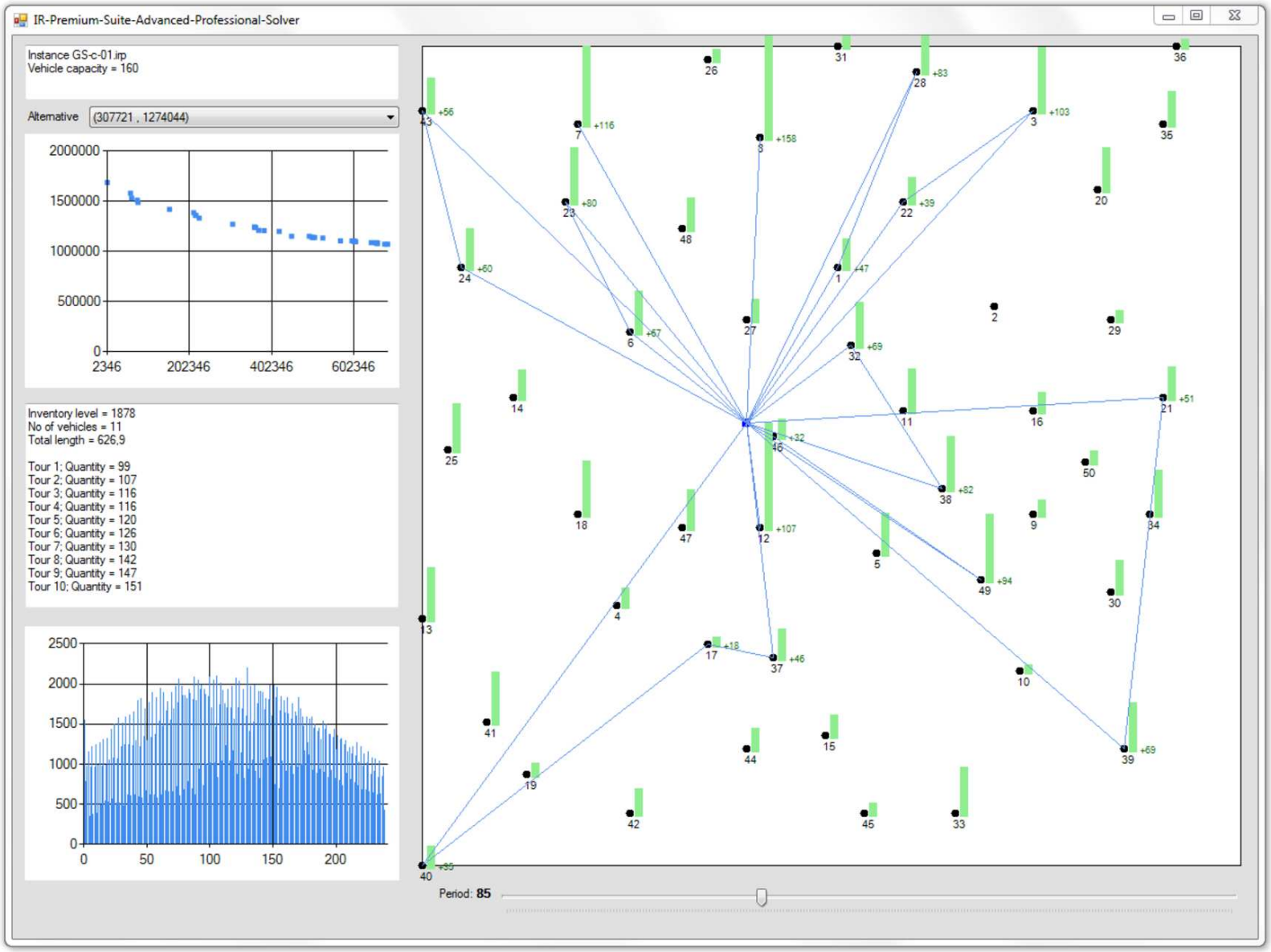

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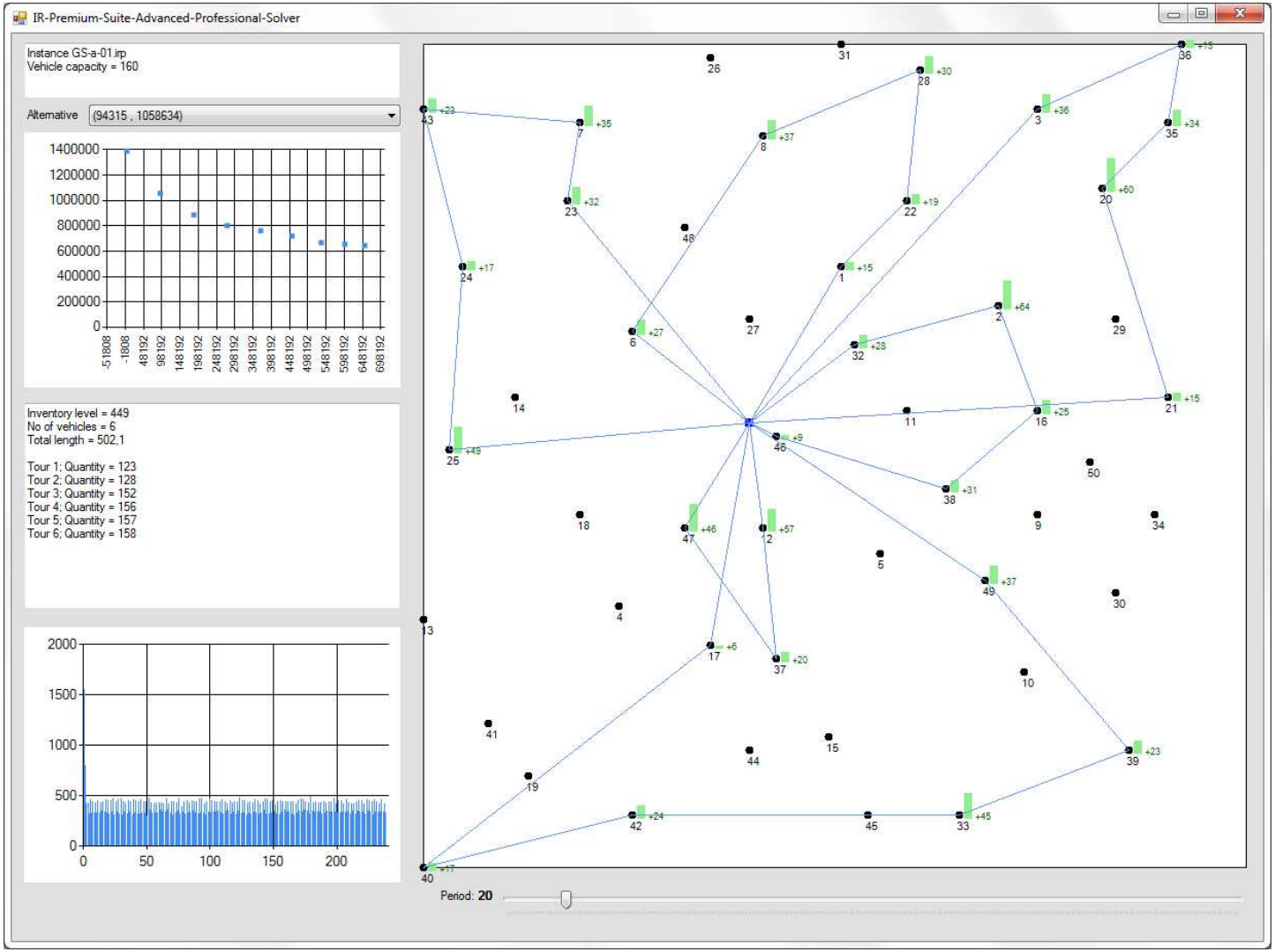

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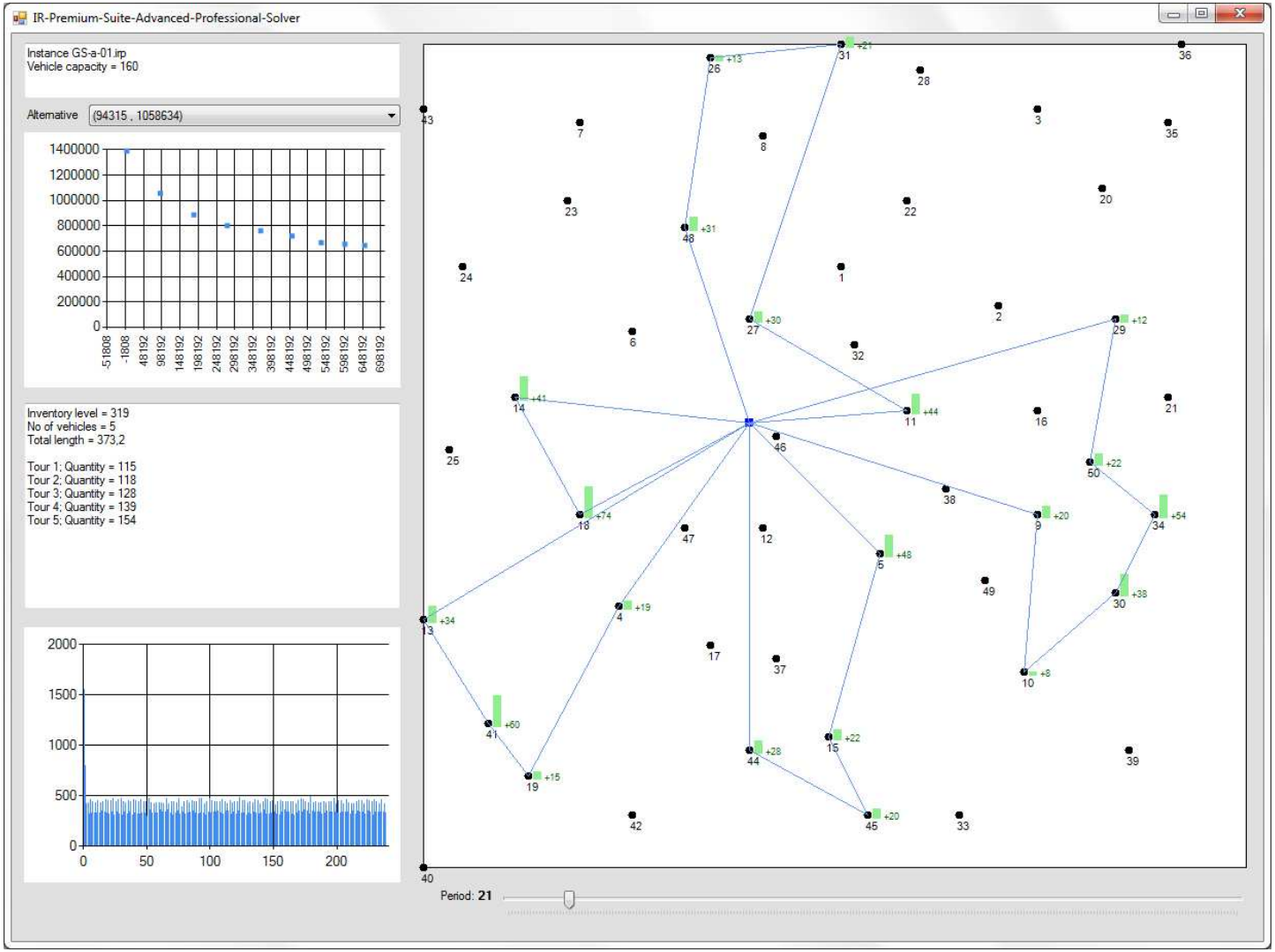

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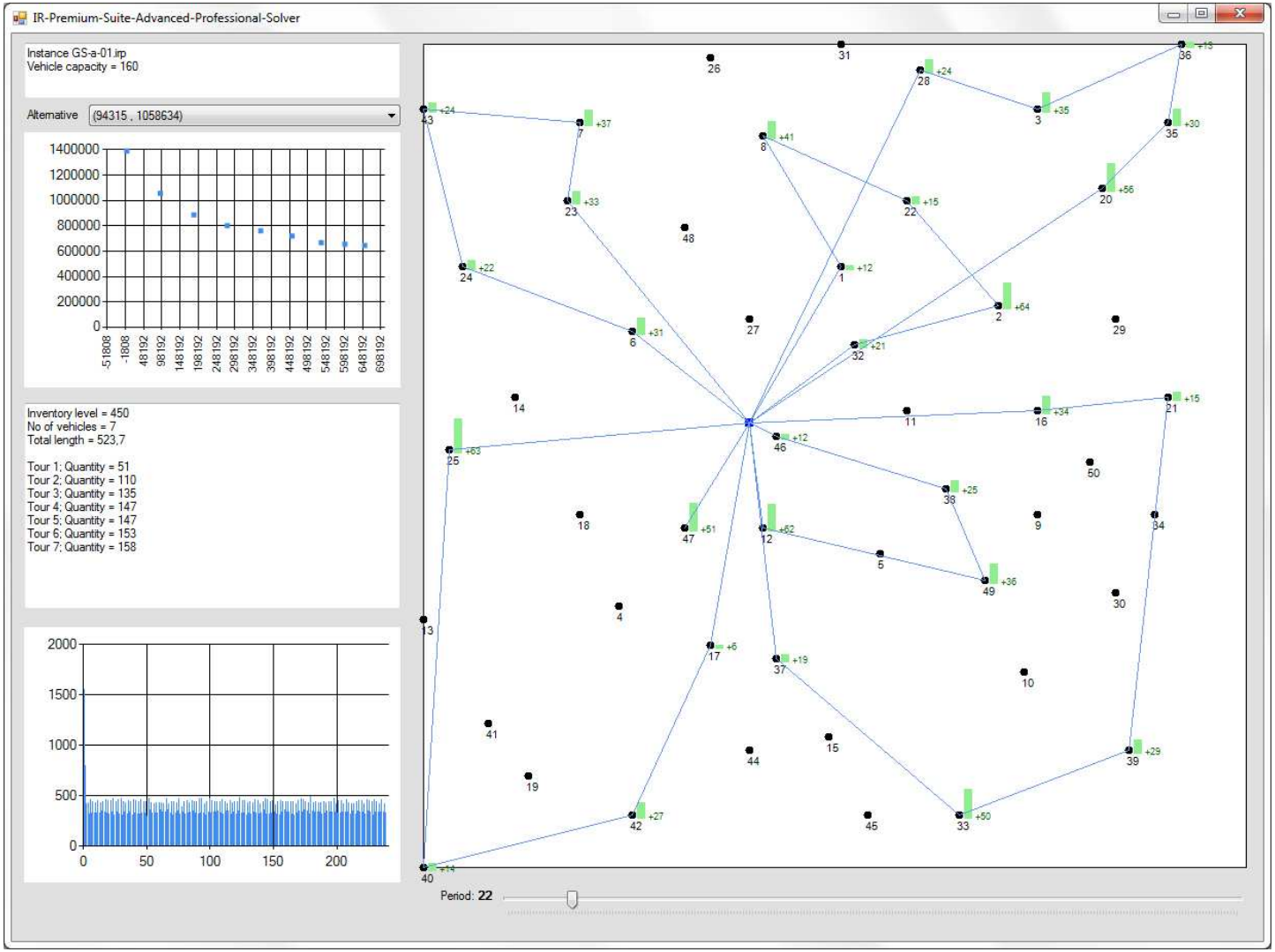

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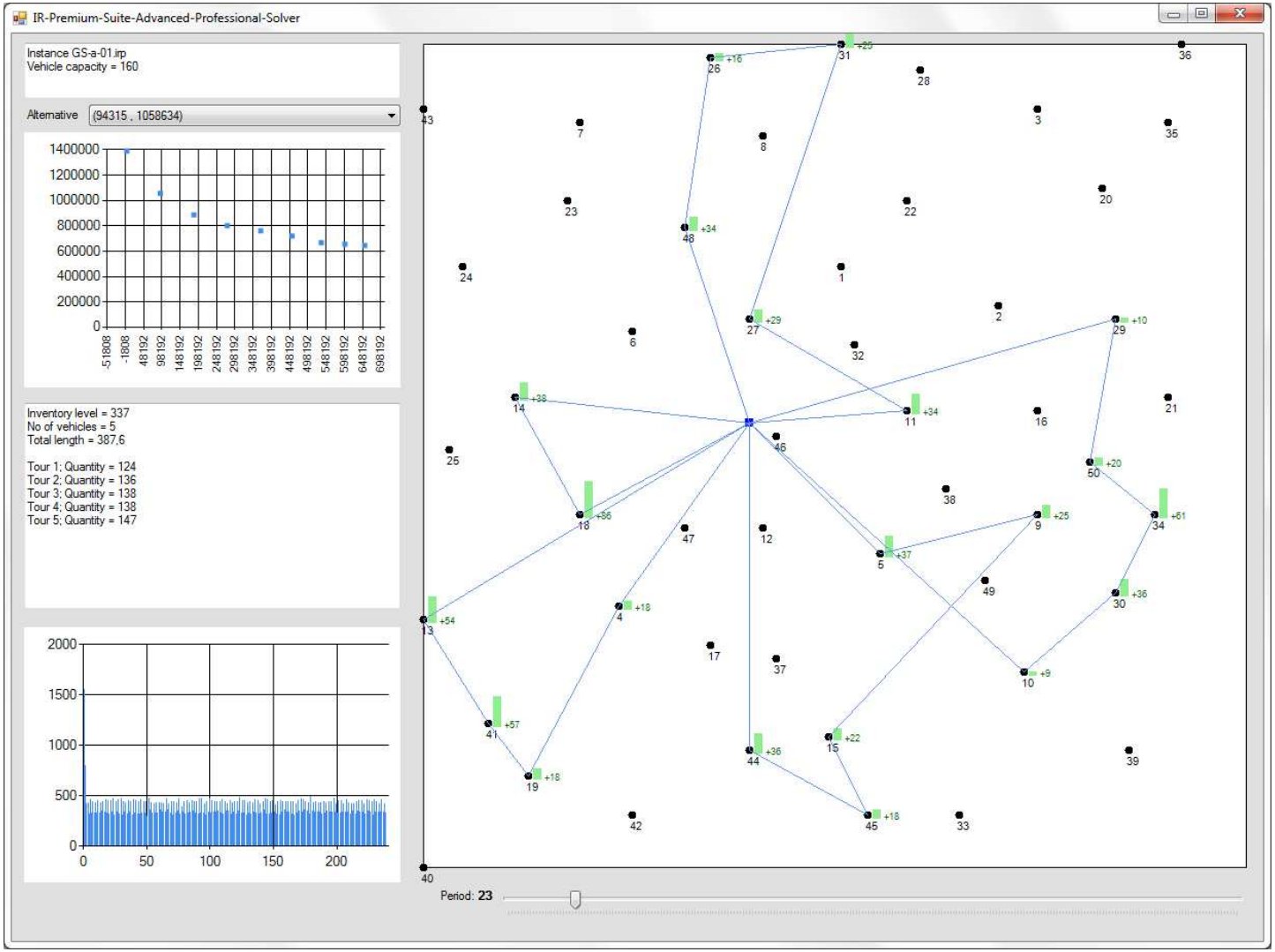

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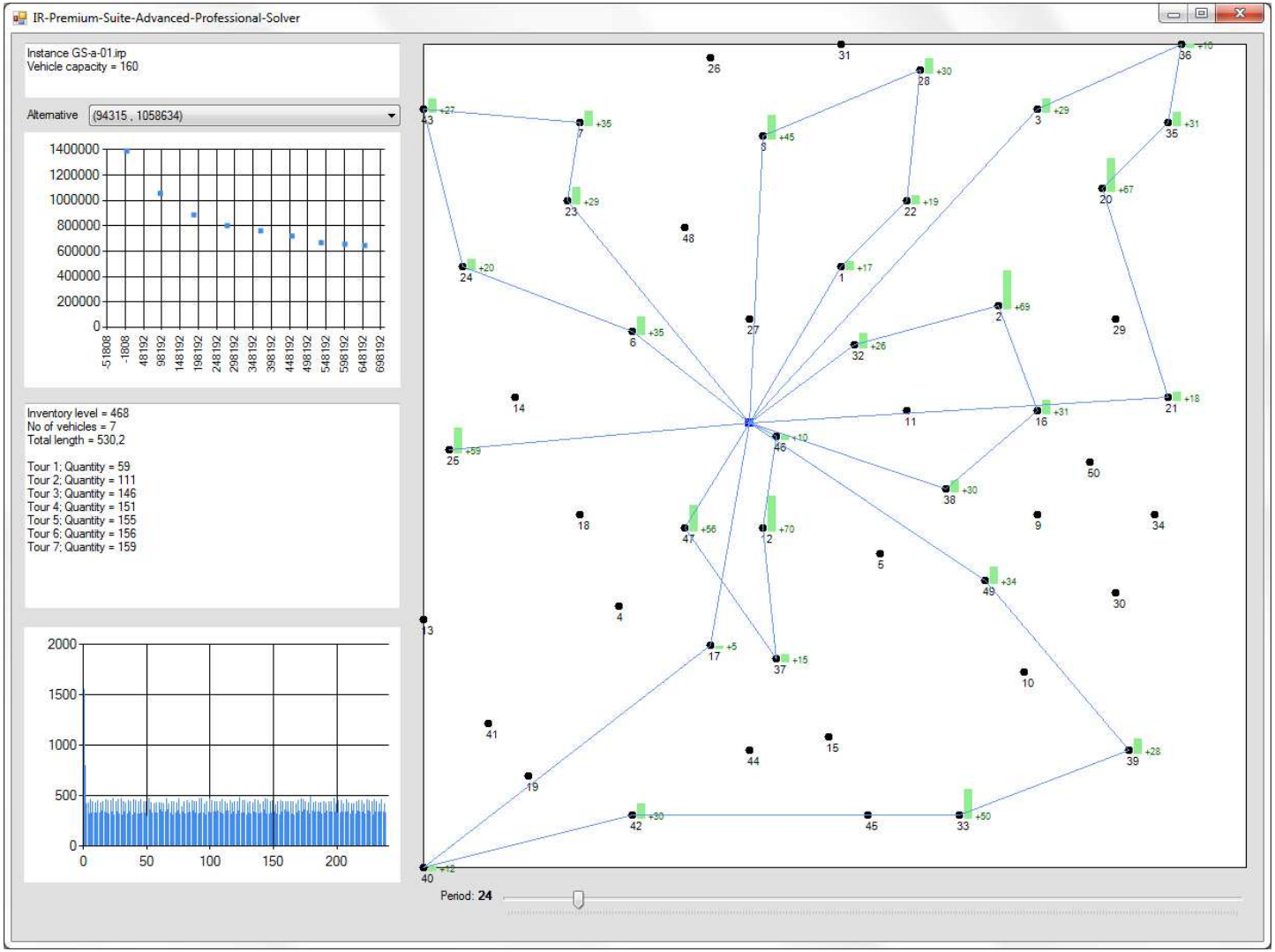

Forthcoming improvements

Already in the box

- The double frequency or binary encoding
  solves the *swing* effect
- Path Relinking
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  Come and see us at MIC 2011 ;-)

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Other improvements should be considered

- improve the routing software
- find new operators (dedicated or combined)
  but be careful with the time for exploration

Forthcoming improvements (cont'd)

In addition, we want to benefit from what we compute

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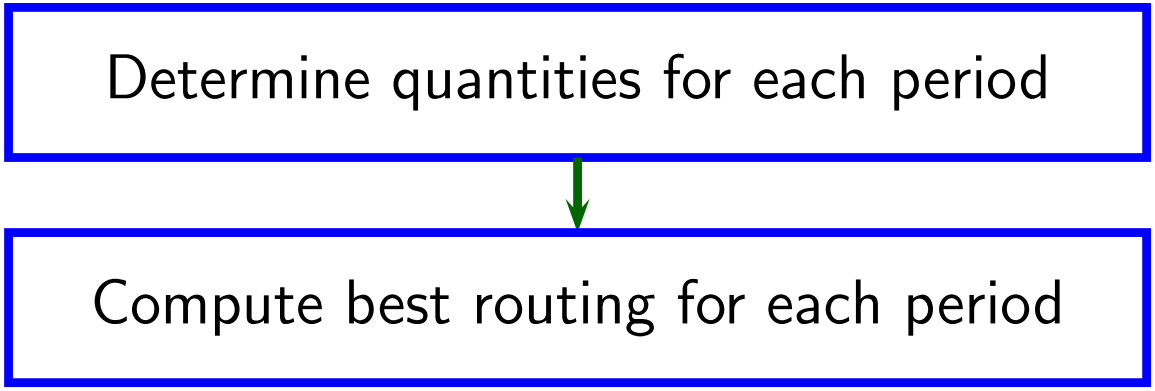

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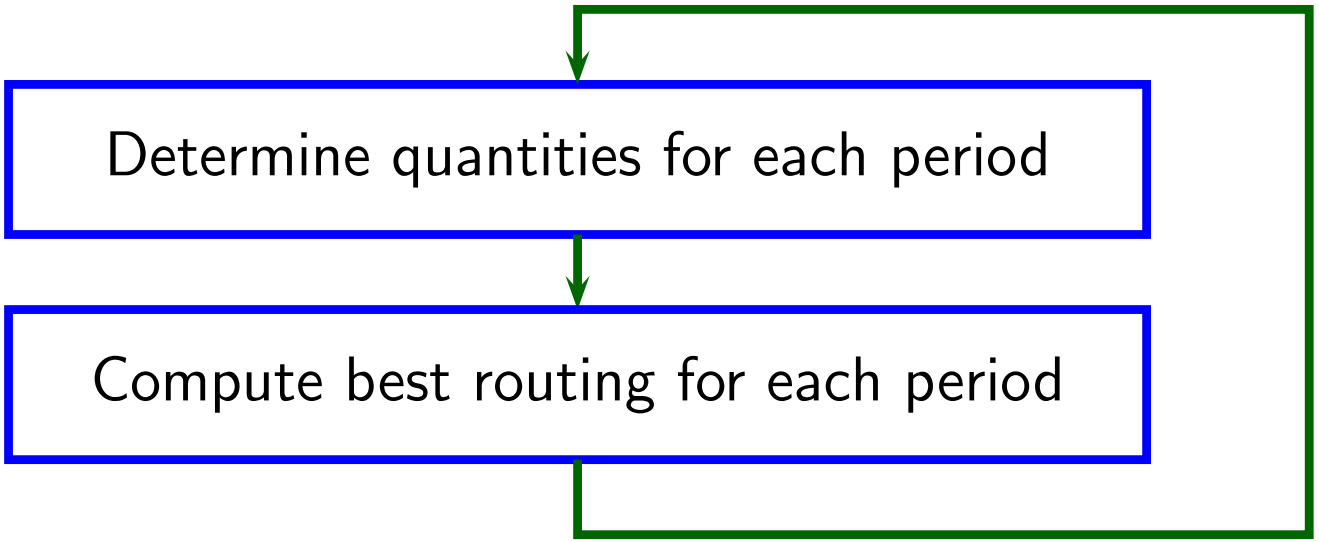

The on-line Inventory Routing Problem

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Answer:

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Statistical *vs* Predictive data

- Company have forecasts (predictive demands)
- At period k, estimation of demand for next 5, 20, 60 days
- We can exploit this information

A first methodology

Data

For each period, we know

- the demand of the current day
- a rough idea for the next 5, 20, 60 days

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A possible methodology would be

- decide of your short term planning horizon
- compute the average demand for it
- solve the problem every period with average values
- cross your fingers that demand will not be too high otherwise send an extra vehicle (at any cost)

But predictions are helpful

Cumulative data for 5, 20, 60 days gives you a tendency

You can use this tendency

- Increasing values
  $\rightarrow$ store more at customer's place (increase stock)
- Decreasing values
  $\rightarrow$ deliver less frequently (for example)

You can also set up a safety stock level

And many more...

Instances for the on-line IRP

A new set of instances has been generated
inspired from Christofides, Mingozzi and Toth VRP instances

Parameters	**Possible values**
# customers, n	$\{50, 75, 100, 120, 150, 200\}$
Horizon, p	$\{240\}$
Truck capacity, K	$\{140, 160, 200\}$
Demand	{Constant, Increasing, Sinus}
Predictions	for each period, cumulative value of the next 5, 20 and 60 periods

This form another new set of 42 instances
available soon at http://logistik.hsu-hh.de/IRP

Visit our web site. . .

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http://www-labsticc.univ-ubs.fr/or/

or the project page

http://logistik.hsu-hh.de/IRP

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