Inventory routing
and on-line inventory routing

Prof Marc Sevaux
marc.sevaux@univ-ubs.fr

Helmut Schmidt University
Logistic Management Dept.
Hamburg, Germany

University of South-Brittany
Lab-STICC, CNRS
Lorient, France

Research Seminar – Magdeburg, Germany
March 20, 2011

Joint work with M.J. Geiger
The Inventory Routing Problem (IRP)

- One depot
- A set of customers
The Inventory Routing Problem (IRP)

- One depot
- A set of customers
- A time horizon
The Inventory Routing Problem (IRP)

- One depot
- A set of customers
- A time horizon
- A known demand for each customer and each period
The Inventory Routing Problem (IRP)

- One depot
- A set of customers
- A time horizon
- A known demand for each customer and each period
- Minimize Inventory cost
The Inventory Routing Problem (IRP)

- One depot
- A set of customers
- A time horizon
- A known demand for each customer and each period
- Minimize Inventory cost
- Minimize Routing cost
The Inventory Routing Problem (IRP)

- One depot
- A set of customers
- A time horizon
- A known demand for each customer and each period
- Minimize **Inventory** cost
- Minimize **Routing** cost

IRP is a **real bi-objective** optimization problem
Important decisions

For solving the IRP, we must make the following decisions:
Important decisions

For solving the IRP, we must make the following decisions:

<table>
<thead>
<tr>
<th>Decisions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. When deliver customers?</td>
<td></td>
</tr>
</tbody>
</table>
Important decisions

For solving the IRP, we must make the following decisions:

Decisions

1. When deliver customers?
2. How much deliver?
Important decisions

For solving the IRP, we must make the following decisions:

**Decisions**

1. When deliver customers?
2. How much deliver?
3. With which routes?
Important decisions

For solving the IRP, we must make the following decisions:

**Decisions**
1. When deliver customers?
2. How much deliver?
3. With which routes?

All these decisions are linked together:
- Increase delivery quantities $\rightarrow$ change routes or frequency
- Change frequency $\rightarrow$ adapt delivery quantities
- . . .
Important references

First papers

[Bell et al., 1983]
Improving the distribution of industrial gases with and on-line computerized routing and scheduling optimizer

[Federgruen and Zipkin, 1984]
A combined vehicle routing and inventory allocation problem

Recent surveys

[Bertazzi et al., 2008]
Inventory routing

[Cordeau et al., 2011]
Short-haul routing
IRP variants

There exists a large number of variants
- planning horizon (finite/infinite)
- inventory costs and capacities
- production/demand rates (of single/multiple product)
- specific restrictions/regulations
- deterministic/stochastic demand/production
- initial inventory
- fleet (homogeneous/heterogeneous)

And different objectives
- usually a combination of inventory level and routing cost
Practical IRP

What does mean **Practical IRP**?

- Main goal: helping companies where IRP is important
- Propose a simple output
What does mean Practical IRP?

- Main goal: helping companies where IRP is important
- Propose a simple output
- Use simple rules (that can be understood)
- Use simple implementation that can be reproduced
What does mean **Practical IRP**?

- Main goal: helping companies where IRP is important
- Propose a simple output
- Use simple rules (that can be understood)
- Use simple implementation that can be reproduced
- Give different alternatives
A major assumption in our work

To follow a common strategy in companies dealing with IRP
We have separated the decisions
A major assumption in our work

To follow a common strategy in companies dealing with IRP
We have separated the decisions

Determine quantities for each period
A major assumption in our work

To follow a common strategy in companies dealing with IRP
We have separated the decisions

Determine quantities for each period

Compute best routing for each period
The choice of policies

Standard policies:

DD  Day-to-day delivery policy
    If not enough in stock, deliver the missing demand
The choice of policies

Standard policies:

DD  Day-to-day delivery policy
    If not enough in stock, deliver the missing demand

OU  Order-up-to level policy
    When you ship, ship the maximum (customer capacity)
The choice of policies

Standard policies:

DD  Day-to-day delivery policy
  If not enough in stock, deliver the missing demand

OU  Order-up-to level policy
  When you ship, ship the maximum (customer capacity)

ML  Maximum level policy (bad name!)
  Any quantity less than the maximum level
  (but which quantity?)
The choice of policies

Standard policies:

DD  Day-to-day delivery policy
    If not enough in stock, deliver the missing demand

OU  Order-up-to level policy
    When you ship, ship the maximum (customer capacity)

ML  Maximum level policy (bad name!)
    Any quantity less than the maximum level
    (but which quantity?)

Assumptions

- Serve only if current inventory is not enough
- If served, only full number of period demands
The frequency policy encoding

Solutions are modeled as a frequency $f$ of the deliveries for each customer

- $f = 1 \rightarrow$ DD policy
- $f = 2 \rightarrow$ serve for the next two consecutive periods
- $\ldots \rightarrow \ldots$
- $f = k \rightarrow$ serve for the next $k$ consecutive periods
- $\ldots \rightarrow \ldots$
- $f = +\infty \rightarrow$ OU policy

The policies are simple to understand (even for companies) and easy to encode for us (vector of integer)
Evaluation of solutions

Each solution is measured with the two criteria

**Inventory cost**

Sum of all inventory levels at customers’ at the end of each period. This can be computed in $O(np)$

**Routing cost**

Sum of all distances run by the trucks at every period. Solve a VRP for each period. This is a $\mathcal{NP}$-hard problem!!!
Evaluation of solutions

Each solution is measured with the two criteria

**Inventory cost**
Sum of all inventory levels at customers’ at the end of each period. This can be computed in $O(np)$

**Routing cost**
Sum of all distances run by the trucks at every period. Solve a VRP for each period. This is a $NP$-hard problem!!!

Routing engines

- Clarke & Wright + savings heuristic
- Record-to-record travel algorithm
Initial solutions

We have implemented several initial solutions
Initial solutions

We have implemented several initial solutions

- **Identical frequency:**
  \( f \) is the same for each customer
Initial solutions

We have implemented several initial solutions

- **Identical frequency:**
  \( f \) is the same for each customer
- **Totally random:**
  \( f \) is chosen at random for each customer
Initial solutions

We have implemented several initial solutions

- **Identical frequency:** $f$ is the same for each customer
- **Totally random:** $f$ is chosen at random for each customer
- **Controlled random:** $f$ is chosen at random between two bounds
Neighborhood

Encoding is a vector of integers
Many moves are possible but think of the neighborhood size

Initial solution

| s | 2 | 3 | 1 | 5 | 1 | ... | 2 |
Neighborhood

Encoding is a vector of integers
Many moves are possible but think of the neighborhood size

Initial solution

\[
\begin{array}{ccccccc}
2 & 3 & 1 & 5 & 1 & \cdots & 2 \\
\end{array}
\]

Move 1: Change frequency by \(\pm 1\) (here \(+1\))

\[
\begin{array}{ccccccc}
2 & 3 & 2 & 5 & 1 & \cdots & 2 \\
\end{array}
\]
Neighborhood

Encoding is a vector of integers
Many moves are possible but think of the neighborhood size

Initial solution

$\begin{align*}
\text{Move 1: Change frequency by } \pm 1 \text{ (here } +1) \\
\text{Move 2: Change frequency by } \pm k \text{ (here } -3) 
\end{align*}$
General algorithm

Algorithm 1: MOEA-IRP

*Initialization:* create an initial population
\[ \rightarrow \text{Identical frequency + controlled random} \]

*Cleanup:* remove dominated solutions

repeat

\[ \text{Improve all solutions with a local search} \]
\[ \rightarrow \text{steepest descent algorithm with Move 1} \]

\[ \text{Rebuild archive with non-dominated solutions only} \]

until no more improvements

Other possible options

- NSGA-II or SPEA
- Path Relinking with elite population
Problem: the *gap* effect

Visualizing the Pareto front, we noticed important gaps:
- how to avoid them?
- how to fill them?
Problem: the *gap* effect

Visualizing the Pareto front, we noticed important gaps

- how to avoid them?
- how to fill them?
Problem: the *gap* effect

Visualizing the Pareto front, we noticed important gaps

- how to avoid them?
- how to fill them?

Potential solutions:

- the *more controlled random* strategy
- the *controlled local search* strategy
Problem: the *swing* effect

In the IRP-solver we developed, we noticed repeating identical patterns

- Customers are grouped together because they run out of goods at the same time
- No clever grouping strategy
- Customers may be served together even if they are far away from each other

Potential solutions:
- the **double frequency** encoding
- the **binary** encoding
Problem: the number of solutions

- This approach generates too many solutions
  → thousands of solutions, how to handle them?

Implemented solution: the reference set strategy
Problem: the *number* of solutions

- This approach generates too many solutions
  → thousands of solutions, how to handle them?

Implemented solution: the *reference set* strategy
Problem: the *number* of solutions

- This approach generates too many solutions
  → thousands of solutions, how to handle them?

Implemented solution: the **reference set** strategy
Numerical experiments

A new set of instances has been generated inspired from Christofides, Mingozzi and Toth VRP instances

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Possible values</th>
</tr>
</thead>
<tbody>
<tr>
<td># customers, $n$</td>
<td>${50, 75, 100, 120, 150, 200}$</td>
</tr>
<tr>
<td>Horizon, $p$</td>
<td>${240}$</td>
</tr>
<tr>
<td>Truck capacity, $K$</td>
<td>${140, 160, 200}$</td>
</tr>
<tr>
<td>Demand</td>
<td>${\text{Constant, Increasing, Sinus}}$</td>
</tr>
</tbody>
</table>

This form a new set of 42 instances available at [http://logistik.hsu-hh.de/IRP](http://logistik.hsu-hh.de/IRP)
Results (too many solutions)

Output for an instance with 50 customers (2485 alternatives)

![Graph showing Inventory-Routing alternatives]
Results (too many solutions)
Results (gaps)

Output for an instance with 50 customers

![Graph showing inventory-routing alternatives with different frequencies in a scatter plot. The total routing cost is plotted against the total inventory cost. The graph includes points for Totally random frq., Controlled random frq., and Identical frq.]
Results (gaps)
Results (improved routing)

Output for an instance with 50 customers

Inventory-Routing alternatives

- Improved controlled freq.
- Improved identical freq.
- Former Pareto front
- New Pareto front

Total routing cost vs. Total inventory cost graph showing different inventory-routing alternatives.
Results (improved routing)
Results (improved routing)
The Inventory Routing Solver
The Inventory Routing Solver (alternatives)
The Inventory Routing Solver (Cost details)
The Inventory Routing Solver (Inventory)
The Inventory Routing Solver (Periods)
The Inventory Routing Solver
Visualization
Vizualization
Vizualization
Vizualization
### Forthcoming improvements

<table>
<thead>
<tr>
<th>Already in the box</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The double frequency or binary encoding solves the <em>swing</em> effect</td>
</tr>
<tr>
<td>• Path Relinking solves the <em>gap</em> problem</td>
</tr>
<tr>
<td>Come and see us at MIC 2011 ;-)</td>
</tr>
</tbody>
</table>
Forthcoming improvements

Already in the box

- The double frequency or binary encoding solves the *swing* effect
- Path Relinking solves the *gap* problem
- Come and see us at MIC 2011 ;-) 

Other improvements should be considered

- improve the routing software
- find new operators (dedicated or combined)
  but be careful with the time for exploration
In addition, we want to benefit from what we compute
In addition, we want to benefit from what we compute

Determine quantities for each period
In addition, we want to benefit from what we compute

- Determine quantities for each period
- Compute best routing for each period
In addition, we want to benefit from what we compute

- Determine quantities for each period
- Compute best routing for each period
Question:
- Can you ask a company what will be its demand next year?

Answer:
The on-line Inventory Routing Problem

**Question:**
– Can you ask a company what will be its demand next year?

**Answer:**
– Yes you can, but the answer will be: *We don’t know*
The on-line Inventory Routing Problem

**Question:**
– Can you ask a company what will be its demand next year?

**Answer:**
– Yes you can, but the answer will be: We don’t know

### Statistical vs Predictive data
- Company have forecasts (predictive demands)
- At period $k$, estimation of demand for next 5, 20, 60 days
- We can exploit this information
A first methodology

Data

For each period, we know

- the demand of the current day
- a rough idea for the next 5, 20, 60 days
A first methodology

Data

For each period, we know

- the demand of the current day
- a rough idea for the next 5, 20, 60 days

A possible methodology would be

- decide of your short term planning horizon
- compute the average demand for it
- solve the problem every period with average values
- cross your fingers that demand will not be too high
  otherwise send an extra vehicle (at any cost)
But predictions are helpful

Cumulative data for 5, 20, 60 days gives you a tendency

You can use this tendency

- Increasing values
  → store more at customer’s place (increase stock)

- Decreasing values
  → deliver less frequently (for example)

You can also set up a safety stock level

And many more...
Instances for the on-line IRP

A new set of instances has been generated inspired from Christofides, Mingozi and Toth VRP instances

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Possible values</th>
</tr>
</thead>
<tbody>
<tr>
<td># customers, $n$</td>
<td>${50, 75, 100, 120, 150, 200}$</td>
</tr>
<tr>
<td>Horizon, $p$</td>
<td>${240}$</td>
</tr>
<tr>
<td>Truck capacity, $K$</td>
<td>${140, 160, 200}$</td>
</tr>
<tr>
<td>Demand</td>
<td>${\text{Constant, Increasing, Sinus}}$</td>
</tr>
<tr>
<td>Predictions</td>
<td>for each period, cumulative value of the next 5, 20 and 60 periods</td>
</tr>
</tbody>
</table>

This form another new set of 42 instances available soon at [http://logistik.hsu-hh.de/IRP](http://logistik.hsu-hh.de/IRP)
Visit our web site...

OR-Group@Lab-STICC
http://www-labsticc.univ-ubs.fr/or/

or the project page
http://logistik.hsu-hh.de/IRP

Contact: marc.sevaux@univ-ubs.fr